Analysis of a Perturbative Transformation Optics-based Spectral-Domain Technique for Field Computation in Tilted Planar-Layered Media

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Summary: We propose and analyze a perturbative technique to model electromagnetic wave propagation and scattering in (locally tilted) planar-layered media by leveraging the theory of Transformation Optics (T.O.) to map the tilted-layered geometry (and fields) to a parallel one. This perturbative technique allows robust *spectral-domain* modeling of electromagnetic sensors in more diverse geophysical formations than strictly parallel-layered ones. A significant challenge consists of quantifying spurious computation error source(s), which fundamentally arise from approximating a transverse translation-*variant* domain with a transverse translation-*invariant* one. Our two main contributions are: (1) Quantifying how said error varies versus the modeled tilting and sensor/environment parameters, and (2) Characterizing sensitivity (to interface tilting) of tensorial (triaxial) sensor responses. Our main results are: (1) Spurious error predominantly scales quadratically with modeled tilt, (2) The sensor's computed measurements show different sensitivities to tilt orientation, suggesting measurement strategies to invert interface tilt.

Background: Numerical eigenfunction expansion ("spectral-domain") techniques advantageously serve modeling needs in diverse applications involving layered media, such as subsurface geophysical exploration and light propagation through layered crystalline structures. Notably, one can cite several benefits of plane wave expansion (PWE)-based modeling of EM radiation in planar-layered media (as compared to, say, Finite Element or Finite Difference methods), such as high computational speed and error-controllability that are *robust* to variations in material properties, layer thicknesses, source radiation frequency, and source position/orientation. However, said numerical spectral-domain methods are restricted with respect to the geometries that can be modeled: for example, PWE require the layer interfaces to be perfectly parallel.

Methodology: In many applications, such as subsurface geophysical exploration, it may be more appropriate to model the sensor's local, surrounding environment as exhibiting *tilted* planar interfaces. A question naturally arises as to how to extend PWE, which are restricted to modeling fields in *parallel* planar layers, to model scenarios involving layers that locally (i.e., in the sensor's vicinity) have planar but *tilted* interfaces. T.O. provides a route for mapping the tilted-layer domain, containing some generally anisotropic and lossy media, to a new computational geometry consisting of parallel planar layers in a perturbative fashion. In particular, each "flattened" interface is coated by T.O.-prescribed, *doubly-anisotropic* "interface-flattening" media that locally mimic the effect of the original, tilted interfaces with general polar and azimuth orientation.

Limitations: This perturbative approach is not without drawbacks, however. Key among them, the doubly-anisotropic "interface-flattening" media are *not* reflectionless, and hence will produce spuriously scattered fields that interfere with the actual fields scattered from the effectively tilted interfaces. This spurious scattering must be quantified versus sensor, environment, and modeled tilting characteristics to understand the practical modeling limitations of this perturbative technique. In this presentation, we discuss the algorithm's theoretical basis, as well as our quantitative study and preliminary modeling results.