A Reexamination of the Single Scattering Approximation

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With the increasing speed of computers and the development of more sophisticated numerical methods, complex electromagnetic problems are being attacked and solved. Yet with this advance, there is the ever present pitfall of losing sight of the fundamental physics involved in the problem. In fact in many cases it may be advisable to use a less accurate approximate solution to get a better understanding of the fundamental interactions involved in the problem. With this as a background, this paper will examine the so-called single scattering or local approximation that is frequently used in very complex problems that can be described by boundary integral equations of the second kind. The single scattering or local approximation amounts to assuming that the unknown current, say, at a point on a rough surface is not dependent upon the current at any other point on the surface. While this is a very simple definition, it is not always easy to pull the single scattering solution out of a second boundary integral equation. For example, in rough surface scattering from the perfectly conducting surface, the Kirchhoff approximation is frequently taken as the single scattering solution and this does work very well for a class of surfaces but not all. However, for another class of surfaces one must employ a boundary perturbation approach to obtain an accurate solution. Interestingly enough this solution obeys the definition of a single scattering or local approximation but seems to require cooperative scattering from two points on the rough surface. In addition, this solution seems to come from the second Neumann iterate of the governing integral equation. This paradox raises the question of what truly is the single scattering or local approximate solution of a second kind boundary integral equation. This difficulty will be addressed through the application of the generalized mean value theorem for integrals as applied to the second kind integral equation. Following the application of this theorem to the second kind integral equation, we show that the local approximation leads to an approximate solution of the form $J_o(x)[1-\int G(x,x')dx']^{-1}$ where $J_o(x)$ is the Kirchhoff term and G(x, x") is the propagator or kernel in the integral equation. We will discuss this solution and our statement that this is *the* single scattering solution. In addition we will consider what happens when an attempt is made to apply the logic developed herein to a preconditioned form of the second kind integral equation such as with the Method of Ordered Multiple Interactions (MOMI). Some attention will be devoted to how one might improve on this true single scattering solution and a physical consequence of the mathematical condition that the kernel G(x, x') be a positive quantity.