Time-domain Solution to Maxwell's Equations for a Lightning Dart Leader and Subsequent Return Stroke

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Lightning detection and geolocation networks have found widespread use by the utility, air traffic control and forestry industries as a means of locating strikes and predicting imminent recurrence. Accurate lightning geolocation requires detecting VLF radio emissions at multiple sites using a distributed sensor network with typical baselines exceeding 150 km, along with precision time of arrival estimation to triangulate the origin of a strike. The trend has been towards increasing network accuracy without increasing sensor density by incorporating precision GPS synchronized clocks and faster front-end signal processing. Because lightning radio waveforms evolve as they propagate over a finitely conducting earth, and that measurements for a given strike may have disparate propagation path lengths, accurate models are required to determine waveform fiducials for precise strike location. The transition between the leader phase and return stroke phase may offer such a fiducial and warrants quantitative modeling to improve strike location accuracy.

The VLF spectrum of the ubiquitous downward negative lightning strike is able to be modeled by the transfer of several Coulombs of negative charge from cloud to ground in a two-step process. The lightning stepped leader ionizes a plasma channel downward from the cloud at a velocity of approximately 0.05c, leaving a column of charge in its path. Upon connection with a streamer, the subsequent return stroke initiates at or near ground level and travels upward at an average but variable velocity of 0.3c. The return stroke neutralizes any negative charge along its path. Subsequent dart leader and return strokes often travel smoothly down the heated channel left by a preceding stroke, lacking the halting motion of the preceding initial stepped leader and initial return stroke. Existing lightning models often neglect the leader current and rely on approximations when solving for the return stroke.

In this study we present an analytic solution to Maxwell's Equations for the lightning leader followed by a novel return stroke model. We model the leader as a downward propagating boxcar function of uniform charge density and constant velocity, and the subsequent return stroke is modeled as an upward propagating boxcar with a time dependent velocity. Charge conservation is applied to ensure self-consistency of the driving current and charge sources, and physical observations are used to support model development. The resulting transient electric and magnetic fields are presented at various distances and delay times and compared with measured waveforms and previously published models.