Scalar Potential Formulation for a Uniaxial Inhomogeneous Medium

Michael J. Havrilla*
Air Force Institute of Technology, WPAFB, OH 45433 USA

Vector potentials are frequently utilized in the electromagnetic analysis of problems involving simple (i.e., linear, homogeneous, and isotropic) media. In recent decades, various scalar and vector potential formulations have been investigated for the analysis of anisotropic and bianisotropic media. This interest has been greatly influenced by the significant developments in material fabrication capability and the phenomena associated with complex media. Uniaxial anisotropic media are particularly interesting from an application viewpoint due to the relative ease of manufacturing this type of material.

The goals of this paper are to first briefly review a scalar potential formulation for a magnetically and electrically uniaxial anisotropic medium. It is assumed the medium is, in general, inhomogeneous along the longitudinal axis (i.e., the *z*-axis). Next, expected and unexpected depolarizing dyad contributions are identified in the scalar potential development. It is discussed, from a physical viewpoint, why the unexpected depolarizing dyad should not exist. Based on this insight, the final goal is to mathematically demonstrate the unexpected depolarizing dyad is actually removable, thus leading to a physically and mathematically consistent theory.

The removal of the unexpected uniaxial depolarizing dyad is first demonstrated for a homogeneous medium using a Green's function spectral domain analysis. It is shown via Fourier transformation that a spectral domain term encountered in the complex-plane analysis appears which exactly cancels the unexpected depolarizing dyad. This review of the depolarizing dyad removal for the homogeneous case is important since the spectral-domain analysis reveals important spatial-domain behavior due to the Fourier transform relationship between the spatial and spectral domains. The insight gained from the consideration of the homogeneous media is subsequently used to demonstrate that the unexpected depolarizing dyad is also removable for the general case of a medium inhomogeneous along the *z*-axis. Demonstrating the unexpected depolarizing dyad is removable is vital to obtaining a mathematically and physically consistent theory and it provides the necessary methodology for correctly handling electromagnetic problems involving complex media.

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