

Scattering by a Half-plane at the Interface Between Isorefractive Media

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In 1896, Sommerfeld obtained a closed-form solution to the scattering of a plane wave by a half-plane, in terms of two Fresnel integrals. That solution is herein extended to the case when the half-plane is located at the planar interface separating two media that are isorefractive to each other.

In terms of rectangular coordinates (x,y,z) , the upper half-space $y>0$ is filled with a linear homogeneous isotropic medium characterized by a wavenumber k and an intrinsic impedance Z_1 . The lower half-space $y<0$ is filled with a linear homogeneous isotropic medium characterized by a wavenumber k and an intrinsic impedance Z_2 . Thus, the two media have the same wavenumber but different intrinsic impedances. The perfectly conducting half-plane is located at $(x\geq 0, y=0)$, and the primary plane wave is obliquely incident from the upper half space in a direction perpendicular to the edge $x=y=0$ of the half-plane. This two-dimensional boundary-value problem is solved exactly and in closed form in the phasor domain for both polarizations of the incident plane wave and with time dependence $\exp(+j\omega t)$.

The problem is solved by observing that each Fresnel integral in Sommerfeld's solution describes a transition across an optical boundary from a zero field on one side to a lateral wave on the other side. In order to revert to that situation in our case, the total field in the upper half-space is considered as the superposition of two fields with different amplitudes, one arising from reflection by an infinite plane and the other by scattering from the half-plane. The field in the lower half-space is written as in Sommerfeld's solution, but with an incident field whose amplitude is equal to the transmission coefficient across the interface between the two isorefractive media. The field in the upper half-space is then matched to the field in the lower half-space by imposing the continuity of the tangential components of the electric and magnetic fields across the boundary $(x\leq 0, y=0)$.

The solution obtained herein yields Sommerfeld's solution in the particular case when the two media are identical ($Z_1=Z_2$). This new canonical solution is the only presently known closed-form solution to the scattering of a plane wave by a half-plane located at the planar interface separating two different media.