Stable semi-analytical computation of fields from arbitrarily-oriented dipoles in cylindrically stratified media

H. Moon and F. L. Teixeira
Electroscience Laboratory
The Ohio State University, Columbus, OH, 43212

The computation of electromagnetic fields due to arbitrarily oriented Hertzian dipoles in cylindrically stratified media has been of great interest in a wide range of applications such as geophysical exploration, fiber optics, and radar cross-section analysis. Analytical expressions can be obtained via Sommerfeld integrals and spectral Green's functions. However, direct numerical computations based on those canonical expressions can lead to underflow and/or overflow problems under finite-precision arithmetic. This well-known problem is related to the behavior of the eigenfunctions (cylindrical Bessel and Hankel functions) for extreme arguments and/or high-orders. Furthermore, convergence problems for the numerical integration of the associated spectral integrals also exist and become more acute when a disparate range of values is to be assumed for the layer thicknesses, medium parameters, source and observation points, and source frequency.

To handle these issues, we introduce a robust algorithm with the following ingredients: (i) range-conditioned cylindrical functions (RCCFs) , together with properly selected subdomains of evaluation, are used in lieu of standard cylindrical functions to achieve stable integrand computation for all frequencies and ranges of physical parameters. (ii) All pre-existing integrand expressions, including generalized reflection are re-expressed using RCCFs to yield stable computation under finite-precision arithmetic. (iii) Two robust integration paths are employed along the complex spectral plane to ensure fast converging numerical integration for all possible scenarios. We illustrate the algorithm for problems of geophysical interest involving layer resistivities varying from 1,000 Ω·m down to about 10^{-8} Ω·m, operating frequencies from 100 MHz down to 0.01 Hz, and varying layer thicknesses.