A Semi-Analytical Vector Spectral Element Method and Its Application to Waveguide Discontinuity Problems

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In many areas such as waveguide discontinuities, integrated optics, and geophysical prospecting we will encounter quasi 2.5D structures, namely 3D structures that are piecewise homogeneous along one direction (the longitudinal direction) but arbitrarily inhomogeneous along the other two directions (the transverse directions). A full wave analysis with the 3D finite element method (FEM) can be used to simulate the electromagnetic wave scatterings by these quasi 2.5D structures; however, the overall efficiency may be quite low since the homogeneity or piecewise homogeneity along the longitudinal direction is not exploited by the conventional 3D FEM.

In this study, a semi-analytical vector spectral element method (SEM) (J. H. Lee, T. Xiao and Q. H. Liu, IEEE MTT, 54, 437-444, 2006) is proposed to solve the vector wave equation with quasi 2.5D structures. The cross section of the structure is discretized by a 2D FEM mesh, and the longitudinal and transverse field components are discretized by same-order scalar spectral elements and mixed-order vector spectral elements, respectively. The spectral elements are special higher order finite elements with the sampling points and quadrature points defined by the Gauss-Lobatto-Legendre (GLL) polynomials. P- and h-refinement as well as spectral accuracy can be easily realized by using the spectral elements for spatial discretization.

After employing spectral elements for the cross section, the semi-analytically discretized system can be transformed into the Hamiltonian space by first eliminating the longitudinal field components, and then a precision integration method (PIM) (W. Zhong, and J. Zhu, J. Num. Meth Comp. Appl., 17, 26-35, 1996) based on the Riccati equation can be used to do the integration along the longitudinal direction and give the stiffness matrix of the whole structure. The PIM can give the “exact” results on the computers, i.e. with only machine round off error, for a quasi 2.5 D structure with arbitrary length along the longitudinal direction, thus can greatly save the number of unknowns. Some numerical examples of waveguide discontinuities will be presented to demonstrate the high accuracy and efficiency of this semi-analytical vector spectral element method.