Axial Scattering by a Concave Parabolic Cylinder Made of DNG Metamaterial

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A parabolic cylindrical surface separates a medium consisting of DNG metamaterial on the concave side from a medium consisting of DPS material on the convex side. The two media have real refractive indexes of opposite sign and the same intrinsic impedance. The incident field consists of a time-harmonic electromagnetic plane wave propagating parallel to the symmetry plane of the parabolic cylinder and perpendicular to its focal axis, and incident on the cylinder from its concave side. The analysis is conducted in phasor domain, for either polarization of the incident field.

The geometrical optics field consists of the incident plane wave on the concave (DNG) side of the cylinder, and of a non-uniform cylindrical wave on the convex (DPS) side of the cylinder, that appears to be emanating from a line source located at the focal axis.

An exact treatment of the boundary-value problem is pursued by introducing parabolic-cylinder coordinates. The problem is thus reduced to the solution of a second-order partial differential equation for the component of either electric or magnetic field parallel to the focal axis. Depending on the ansatz made as to the form of the solution of this equation, the result will be expressed in terms of a parabolic-cylinder function, or a complementary error function, or a Fresnel integral, all these forms being ultimately equivalent to one another. The ansatz adopter herein is that used by Lamb in studying the reflection by a perfectly conducting parabolic mirror (H. Lamb, *Proc. London Math. Soc.*, vol. 4, pp. 190-203, March 1906), resulting in an exact solution expressed as the product of an exponential times a Fresnel integral. This exact solution is compared to the geometrical optics solution.