Imaging Effect of an Elliptic Perfect Lens

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Imaging an isotropic line source to an anisotropic line source from one focal to another focal point of a perfect elliptic lens is investigated. Geometrical solution in the optical limit is verified by an analytical and numerical solution of electromagnetic behavior of an elliptical lens. Mathieu functions that require a negative value of a real parameter are used to obtain the solutions.

The topic of complex materials in which both permittivity and permeability have negative values at some frequencies has attracted considerable attention in recent years. In this study, the effect of an elliptic cylinder made of double negative (DNG) metamaterial on an isotropic line source located at one of the focal lines of the cylinder is considered. It is assumed that the cylinder has real and negative refractive index. The medium surrounding the cylinder is double-positive (DPS) having real but positive refractive index. However, both media have the same intrinsic impedance value.

In a previous work, it was shown that in the optical limit the optical line source is imaged to a non-isotropic line source located at the other focal line. This virtual line source has a radiation pattern with maximum power radiated in the direction from the virtual line source to the real one and minimum power radiated in the opposite direction. In this work we will verify that the electromagnetic behavior of the perfect elliptical lens has the same properties with the optical behavior when we increase the frequency.

The electromagnetic fields inside and outside the lens are expressed in terms of infinite series of elliptic-cylinder wave functions, involving products of radial and angular Mathieu functions. Since the wavenumbers inside and outside the lens have opposite signs, it is not possible to enforce the boundary conditions at the surface of the lens on a mode to mode basis that would allow an analytic determination of the modal expansion coefficients. Instead, the determination of the coefficients requires the inversion of an infinite matrix. An approximate solution is obtained by truncating this matrix, i.e., by considering only a finite number of terms in the field expansions. Then Galerkin technique is applied to take advantage of orthogonality properties of Mathieu functions to reduce the number of non-zero entries in the truncated matrix equation.

The radiation pattern of the lens is plotted at different frequencies, and considerations are developed on how rapidly the optical pattern obtained is approximated as the frequency increases.