Unconditionally Stable Complex-Envelope FDTD Algorithm for the Analysis of Logging Tools for Borehole Prospection

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Oilfield exploration routinely relies on the use of electrical logging-tools to estimate the resistivity of the earth formation. The finite-difference time-domain (FDTD) method can be applied in the numerical simulation of the response of electric borehole logging-tools – such as logging while-drilling (LWD) tools – in complex earth formations. Among the advantages of FDTD are its conceptual and implementation simplicity, its flexibility to handle complex media, and its matrix-free characteristics. Despite these advantages, FDTD is not efficient to simulate very electrically small structures because the maximum time step size limited by the Courant-Friedrich-Levy (CFL) condition, often resulting in excessively long computation times.

The CFL constraint can be overcome by applying alternating-direction-implicit techniques (ADI) to FDTD [T. Namiki and K. Itoh, *IEEE Trans. Microwave Theory Tech.*, pp. 2003-2007, 1999]. In ADI-FDTD, the time step size can be chosen without stability constraints, being limited only by accuracy considerations. The ADI-FDTD method was later extended to cylindrical grids in [C. Yuan and Z. Chen, *IEEE Trans. Microwave Theory Tech.*, pp. 2401-2405, 2002]. Under same accuracy criteria, the ADI-FDTD method is not significantly faster than the FDTD method due to splitting errors and numerical dispersion errors unless very fine grids are used.

To improve the accuracy of ADI-FDTD for narrowband problems, a complex enveloped (CE) extension of ADI-FDTD was proposed (for Cartesian grids) in [C. Ma and Z. Chen, *IEEE Trans. Antennas Propag.*, pp. 971-975, 2005]. CE ADI-FDTD computes the slowly varying envelope of the field and yields less dispersion error at the carrier frequency than conventional ADI-FDTD, making feasible the use of much larger time steps for same accuracy.

In this work, the CE ADI-FDTD algorithm is extended to the 3-D cylindrical grids and used to simulate the responses of LWD tools in earth formations. The cylindrical grid comforms to the geometry of LWD tools and avoids staircasing (discretization) errors. The proposed cylindrical 3-D CE ADI-FDTD method also utilizes non-uniform gridding, which lead to a reduced number of grids points for a given physical domain size. An envelope ramped sinusoidal excitation as the source for a 2MHz carrier frequency. The 3-D cylindrical CE ADI-FDTD method is validated against FDTD and numerical mode matching (NMM) results [Y.K. Hue and F.L. Teixeira, *IEEE Trans. Geosci. Remote Sens.*, pp. 257-268, 2005] in a uniform formation for various formation conductivity and [Y.K. Hue and F.L. Teixeira, *IEEE Trans. Geosci. Remote Sens.*, pp. 78-81, 2005] crossing a three layer formation with a dipping bed, respectively. The comparisons show good agreement. (Acknowledgment: This work was supported in part by NSF and OSC).