On the Application of the Continuum Model of Coupled Oscillator Arrays with Coupling Delay to Finite Arrays

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The concept of using arrays of mutually injection locked voltage controlled oscillators to feed the elements of a phased array antenna, originally proposed by Liao and York [IEEE Trans. MTT-41, 1810-1815, October 1993], has been studied extensively in the intervening years. One of the formulations that has proved of significant conceptual utility is the so-called continuum model.[Pogorzelski, et al., MTT-47, 463-470, April 1999] Recent theoretical studies have been concerned with accounting for coupling delay in the continuum model to render the theory causal.[Pogorzelski, URSI/EMTS Digest, Ottawa, Canada, 2007] However, the work reported to date has only been applied to infinite one dimensional arrays. In such arrays, a steady state is never reached. Practically speaking, arrays are always finite in size so the continuum model must be extended to accommodate a finite size array. The present paper reports such an extension.

The approach to extending the continuum model is based on the exact solution of the discrete model for finite arrays.[Pogorzelski, URSI/NARSM Digest, Ottawa, Canada, 2007] In developing that model, an equivalent reflection coefficient for the array ends was derived in terms of the exponential delay factor in the Laplace domain and a "propagator" in terms of an arcsech that carries one from one oscillator to the next in the array. This exact propagator is approximated in the continuum model by means of a square root which is approximately equal to the arcsech in the vicinity of the branch points. This square root approximation is then used to obtain an approximate reflection coefficient for the array ends in the continuum model and this reflection coefficient is in turn used to obtain the solution for a finite array from that of an infinite array.

A current drawback of the continuum model including coupling delay is the complexity of the inverse Laplace transform integration performed along the branch cuts. These branch-cut integrations can be computed numerically but that diminishes the utility of the solution as a means of conceptually understanding the array dynamics. One might also consider reverting to the discrete model for which an exact solution is known. However, the discrete model yields a solution in the form of a finite series for each oscillator in which the number of terms in the series increases with time and becomes infinite in steady state, a most inconvenient circumstance. Thus, some thought has been given to asymptotic evaluation of the inversion integral in the continuum model. Such an evaluation has so-far yielded quite satisfactory results for a single detuned oscillator in an infinite array. These will be presented together with a discussion of the more general theoretical effort in this direction aimed at obtaining an asymptotic result for a finite array with detuned end elements such as is used in an agile beam antenna.