Maximum Recoverable Energy from a One Port Network and its Relationship to Bandwidth

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An important property of the Q factor (quality factor) in oscillating systems has always been to determine something about the system bandwidth. Provided that the bandwidth of the system is "small enough," Q and bandwidth are for practical purposes inversely proportional. Many sources also express Q in the form $Q = \omega U/P$, where ω is the resonant frequency of the system, U is the (time-averaged) stored energy and P is the power lost to the system. Even if this is understood as only an approximation, this relationship is in a fundamental way inconsistent with the way bandwidth and Q are related at the system's input terminals. Bandwidth is uniquely determined by behavior at the input terminals, while stored energy is not.

An extreme example of this is a lossless transmission line terminated in its characteristic impedance. At the input terminals the impedance seen is a frequency-independent resistance, so the bandwidth is interpreted to be infinite. The stored energy, on the other hand, is proportional to the length of the transmission line, and could be made arbitrarily large by making the line long enough. This contradicts the purported inverse relationship between Q as defined above and bandwidth, raising the question of how one can legitimately compute this quantity merely from a knowledge of the stored energy and power lost. This is a particularly important issue for calculation of the bandwidth of an antenna, where the stored energy is known to be infinite. Various ways to get around this difficulty have been proposed in the past, but none has led to a unique resolution of the problem. In particular, the pioneering work of Chu (1948) used the energy in an equivalent circuit for the antenna to compute the Q, but since there are other equivalent circuits that store more energy, it is not clear that this approach yields the correct "bandwidth Q".

We propose a possible solution to this problem for the case of a one-port network using the concept of Maximum Recoverable Energy (MRE). This is defined as the maximum amount of energy that may be extracted from the network at its terminals. This may be done by using an appropriate optimal choice of time-dependence for the source attached to the port. Only the terminal behavior of the system is needed to determine the MRE, which has been found to be unique. We suggest using the MRE rather than actual stored energy in the definition of Q in order to obtain a quantity that is accurately related to bandwidth. Examples involving simple circuits will be presented to illustrate the difference between Qs calculated by these various methods.