Investigation of Fourth Order FDTD Method Using Symmetric Boundary Extrapolation

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Maxwell's Equations are solved using a long-stencil fourth order finite difference time domain (FTDT) method over a Yee grid. The derived scheme is shown to be fourth order accurate in both space and time. Boundaries are handled using a symmetric extrapolation method that leads to ghost grid points around the physical boundary, ensuring that the results maintain fourth order accuracy and conditional stability. The method is demonstrated and analyzed for two canonical problems in electromagnetics, the rectangular cavity and the ridge-loaded cavity.

Since Yee's pioneering work on the FDTD method over a staggered orthogonal grid (Yee 1966), many advances and improvements to the method have been attempted. One such improvement is the extension of Yee's original method (which is second order accurate) to a fourth order scheme that could reduce numerical dispersion and produce better results for late-time analysis (J. Fang 1989). Fourth order schemes also have an advantage in that they often allow a much courser grid to be employed than a second order scheme.

The boundary conditions, however, often present a challenge to fourth order FDTD methods. A direct application of a fourth order long-stencil difference results in a one-sided approximation formula around the boundary, such as a fourth order closure proposed by P. Petropoulous and A. Yefet in 2001. Another approach is to introduce a compact difference operator for the spatial derivatives over the Yee stencil (Young 1997). Unfortunately, these approaches tend to either suffer from instability near the boundary or require an implicit updating scheme (at a high computational cost).

In the present work, the developed method is fully explicit. The only added computational step is the assigning of the symmetric ghost point values around the boundary. Such an extrapolation method may appear to be only second order accurate, but careful analysis shows that it is fourth order accurate, preserving the fourth order character of the overall scheme. This extrapolation method does not suffer from instability near the boundary. Additionally, the time stepping method used by most 4th order FDTD schemes (the RK4 integrator) is improved upon. The RK4 integrator is replaced by Jameson's method, a four stage integrator requiring fewer temporary storage variables than RK4 and saving memory. The resultant scheme is conditionally stable, having a CFL number of $\frac{12\sqrt{2}}{7}$.

Finally, this method is tested for the rectangular cavity and the ridge-loaded cavity. The simulation results are analyzed both in the time domain to rigorously establish fourth order convergence, and in the frequency domain to establish the accuracy of the resonant frequency computations.