Path Integrals in Electromagnetics

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In searching for various representations of the electromagnetic fields, some recent interest has been placed on the use of path integrals. These in some sense propagate fields from previous times to some time of interest by all possible spatial paths appropriate to the time difference to the present. This paper reviews some standard formulations and compares them to the path-integral formulation to give an explicit interpretation for electromagnetics.

In representing the solution to electromagnetic problems (e.g., radiation, scattering, propagation) in various media, there are various possible approaches. One can solve differential equations appropriate to special geometries (e.g., waveguides, cavities). One can formulate integral equations over antennas and scatterers using appropriate Green functions, usually of the free-space variety. One can discretize the Maxwell equations in frequency and time domains for a numerical (computer) solution to geometries and media with intractable analytic solutions. Symmetries can also be used to reduce the solution difficulty. The above can be elaborated in various ways using various mathematical techniques.

Another potential approach involves the path integral introduced by Feynman. This has been introduced into electromagnetics. Intuitively, the path integral represents the electromagnetic fields at some point in space by integrals over other portions of space along all possible paths between each coordinate, to the position of interest. Of course, causality can be invoked to limit the paths of integration to previous times, and to regions of space where there are nonzero fields and sources within a transit time on a path.

The path integral provides a way to integrate over fields from previous times (initial conditions) to find the fields at later times. This can also include integrals over sources. As the initial time tends to $-\infty$, the usual Green function integral over sources is left. As an added benefit we have found a special form of inhomogeneous/anisotropic medium for which the problem reduces from 6×6 matrices to 3×3 matrices.

These representations can also be compared to that from the field equivalence theorem. In this form the fields and sources in a volume can be replaced by equivalent electric and magnetic sheet currents on the boundary surface, or equivalently the tangential fields. While the path integral is based on initial conditions, the equivalence principle is based on boundary conditions.