

# Surface Current Second-Order Differential Equation

## For a Radiation Problem

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**Abstract**– For a radiation problem rather than a scattering problem, the technique of measured equation of invariance (MEI) is used to convert a MoM full matrix to a tri-diagonal matrix with an unchanged excitation vector. From a finite difference point of view, the tri-diagonal matrix is equivalent to the discrete form of the finite difference (FD) for a second-order differential equation. Thus, the surface current second-order differential equation for a radiation problem like a thin wire antenna can be obtained from its corresponding tri-diagonal matrix. A mathematical significance of this kind of transformations is that for an arbitrarily shaped wire antenna, there equivalently exists a lowest-order (the 2<sup>nd</sup> order) surface current differential equation. In other words, for a radiation problem, there exists the relationship between a surface integral equation and an equivalent second-order differential equation. Furthermore, the physical meaning of this kind of transformations is that a global relationship between discrete nodes could be simplified into a local relationship between just neighboring nodes. Finally, the computation speed to solve a tri-diagonal matrix generated by the 2<sup>nd</sup> order differential equation is much faster than to solve a full matrix generated by the integral equation. The numerical examples for a straight dipole and a loop antenna show that the results obtained by the 2<sup>nd</sup> order differential equation remains the same accuracy as the results obtained by the integral equation.