A New Criterion for DOA Estimation of Coherent Sources Based on Weighted Spatial Smoothing

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1. INTRODUCTION

Subspace-based array signal processing techniques such as MUSIC for bearing estimation find wide applications in a variety of fields ranging from radar, sonar, oceanography and seismology to radio astronomy. Their super-resolution capabilities are severely degraded, however, when coherent signals are presented. Spatial smoothing (SS) is one of the common approaches to this problem, which was first proposed by Evans et al [1] and later developed by Shan et al [2], Williams et al [3], Pillai and Kwon [4]. Unfortunately, this preprocessing scheme always results in an inferior resolving ability for closely-spaced coherent signals. On the one hand, SS decreases the effective aperture of the array and on the other hand, the smoothed source covariance matrix is just a full rank and not a diagonal matrix, which means that the remaining degree of correlation between sources will still exert unfavorable effect on the following subspace-based algorithms.

In order to decorrelate the coherent sources perfectly, i.e. recover the smoothed source covariance matrix to a diagonal matrix, in [5] Wang et al proposed a weighted spatial smoothing (WSS) preprocessing scheme. Based on the idea underlying WSS, a new criterion was proposed in this paper for the direction-of-arrival (DOA) estimation of coherent sources. Compared with the WSS in [5], the cost function constructed from the new criterion can achieve favorable DOA estimation of coherent sources without the prior knowledge of source direction and the de-noising preprocessing. In essence, the proposed criterion is no longer a preprocessing scheme since no following subspace-based algorithms are needed.

2. DATA MODEL

Consider M narrowband plane waves, from directions $\theta = [\theta_1, \cdots, \theta_M]^T$ and centered at frequency ω_0, impinging on an uniformly linear array composed of N omnidirectional sensors and separated by a distance d. This scenario can be described by the following data model (1):

$$X(k) = A(\theta)S(k) + N(k) \tag{1}$$

where $X(k)$ is a noise-corrupted array output vector, $S(k)$ is a N×1 signal vector, $N(k)$ is a N×1 noise vector. Array manifold matrix $A(\theta) = [a(\theta_1), \cdots, a(\theta_M)]$ is N×M matrix whose columns are the steering vectors defined by (2)

$$a(\theta_k) = \left[1, e^{j\beta_k}, \cdots e^{j(N-1)\beta_k}\right]^T \tag{2}$$

In (2) β_k denotes the wavenumber of kth source and is expressed by (3)

$$\beta_k = \frac{2\pi}{\lambda_0} d \sin(\theta_k) \tag{3}$$

The $N \times N$ array covariance matrix R is defined by (4)

$$R = E\left[X(k)X^H(k)\right] = AR_S A^H + \sigma^2 I \tag{4}$$

where $R_S = E\left[S(k)S^H(k)\right]$ is M×M source covariance matrix. The nonsingularity of the R_S is the key to successful applications of MUSIC-like methods. In our formulation, superscripts T and H denote transposition and conjugate transposition, respectively. E[·] denotes the statistical expectation and I is $N \times N$ identity matrix.

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3. SS TECHNIQUE AND WSS TECHNIQUE

The underlying basis for MUSIK-like algorithms is the orthogonality between the noise subspace and signal subspace of array covariance matrix R. However, if signals are coherent each other, the source covariance matrix R_S becomes singular so that some of its eigenvalues are zero. This means that part of the signal subspace is indistinguishable from the noise subspace and results in a divergence of signal eigenvectors into noise subspace. As a result, the observed noise subspace is no longer orthogonal to the steering vectors in the matrix A and MUSIC algorithm fails.

The basic idea underlying SS is to split the main array into a number of overlapping subarrays, then the subarray covariance matrices are averaged. The spatial smoothing induces a random phase modulation which in turn tends to decorrelate the signals that caused the rank deficiency. let m denote the size of subarray, implying that the number of subarrays is L=N-m+1, then a compact expression for spatial smoothedarray covariance matrix R_f can be written as (5)

$$R_f = \frac{1}{L}\sum_{k=1}^{L} F_k R F_k^T \tag{5}$$

with $F_k = \left[0_{m\times(k-1)} \mid I_m \mid 0_{m\times(N-k-m+1)}\right]$.It is generally believed that SS just recovers R_S from rank one to rank M and it is impossible to achieve a diagonal smoothed source covariance matrix by SS. To generate a diagonal smoothed sources covariance matrix, in [5], A weighted spatial smoothing (WSS) technique was proposed, in which not only auto-correlation but also cross-correlation information of subarray outputs is taken into consideration and a weighted sum of all sub-matrixes of array covariance matrix is made to form a smoothed array covariance matrix R'_w . A compact expression of R'_w can be given as (6)

$$R'_w = \sum_{i=1}^{L}\sum_{j=1}^{L} F_i R F_j^H w_{ij} \tag{6}$$

If a weight matrix W is defined as (7)

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1L} \\ w_{21} & \ddots & \ddots & w_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ w_{L1} & w_{L2} & \cdots & w_{LL} \end{bmatrix} \tag{7}$$

a more meaningful expression for R'_w can be written as (8-12)

$$R'_w = A_m R'_S A_m^H + Q'_N \tag{8}$$

$$R'_S = R_S \bullet \left(B^H W B\right) \tag{9}$$

$$Q'_N = \sum_{i=1}^{L}\sum_{j=1}^{L} F_i F_j^H w_{ij} \tag{10}$$

$$B = \left[b_1\, b_2 \cdots b_M\right] \tag{11}$$

$$b_k = \left[1 \quad e^{-j\beta_k} \quad e^{-j2\beta_k} \quad \cdots \quad e^{-j(L-1)\beta_k}\right]^T \tag{12}$$

where R'_S and Q'_N denote the smoothed source and noise covariance matrix after WSS respectively." $\bullet$ " denotes the Hadamard product, i.e. element-wise multiplication of matrix.

In [5] an optimal weight matrix subject to a diagonal R'_S is chosen as (13)

$$W = \left(BB^H\right)^+ \tag{13}$$

where superscript " + "denotes the Moore-Penrose pseudo-inverse of matrix.

4. NEW CRITERION FOR DOA ESTIMATION OF COHERENT SOURCES

Due to he fact that the optimal weight matrix in (13) is a function matrix of source directions, the WSS in [5] assumed that certain priori knowledge about the sources directions is available when constructing the optimal weight matrix. Besides, a de-noising preprocessing is needed to mitigate the effect of the WSS on the array noise. Based on the basic idea underlying the WSS, a new criterion will be formulated as follows to relax above two restrictions of WSS in [5].

It is well known that in the scenario of dependent sources and ideal array "white" noise with covariance matrix $\sigma^2 I$, the array output covariance matrix R is toeplitz. With WSS and weight matrix W as (13), we can de-correlate coherent sources completely to dependent sources. Moreover, although a troublesome "colored" array noise has been introduced after WSS, fortunately, Q'_N always possesses a toeplitz form since the $F_i F_j^H$ is toeplitz and the weighted sum of $F_i F_j^H$ $i = 1 \cdots L, j = 1 \cdots L$ is also toeplitz. So we can conclude that

"After Weighted Spatial Smoothing with weight matrix as (13), the resultant R'_w is always toeplitz."

Based on the criterion above, a cost function (14) can be constructed for effective DOA estimation of coherent sources through the toeplitz matrix fitting of R'_w.

$$\hat{\theta} = \min_{\theta} \left\| R'_w(\theta) - R_T(\theta) \right\|_F^2 \tag{14}$$

$$R'_w(\theta) = \sum_{i=1}^{L} \sum_{j=1}^{L} F_i R F_j^H w_{ij}(\theta) \tag{15}$$

$$W = \left(B(\theta) B(\theta)^H \right)^+ \tag{16}$$

$$R'_T(\theta) = \text{toeplitz}([r_1, r_2, \cdots r_m]) \tag{17}$$

$$r_i = \frac{1}{m - i + 1} \sum_{p=1}^{m-i+1} [R'_w(\theta)]_{p,p+i-1} \quad i = 1,2,\cdots m \tag{18}$$

where R_T is the corresponding toeplitz form of R'_w and $\|\cdot\|_F$ denotes the Frobenius matrix norm.

In the algorithm developed, GA (genetic algorithm) is used as the optimizer .The initial population in GA consists of some random initial estimates of DOAs within the FOV (field of view) of array and the fitness of an individual is inversely proportional to the deviation of R'_w from its corresponding toeplitz form. It is obvious that as the stochastic search of GA continues, the fittest individual in successive generation will converge to the real DOAs of coherent sources and the fittest individual in final generation will be chosen as the final estimates of DOAs.

5. SIMULATION RESULTS

Simulations are carried out for a 6 sensors uniform linear array with one half-wavelength inter-sensor spacing. Two narrowband coherent sources with equal power impinge on the array, from the far filed, at distinct directions $35°$ and $40°$ w.r.t the broadside of array. For all cases 200 snapshots are used to estimate the array covariance matrices R. The number of source is assumed known. With SNR=20dB and L=3, SS is completely disabled in this difficult and rigorous scenario. As depicted in figure 1,we can only get a false spectral peak at about $37.5°$ in all runs. In contrast, the new algorithm can always precisely resolve these two closed-spaced sources easily and the performance is robust very much. For demonstration, figure (2-4) show the result of one run of the new algorithm. Figure (2) shows the initial population distribution (population size is 100), figure (3) shows the final population distribution and figure (4) shows the convergence of the fittest individual (bearing estimates) corresponding to each successive generation.

From the results presented above we can reach a conclusion that the resolving performance of new algorithm for closely-spaced coherent sources is highly superior to that of SS algorithm and the size of array required by new algorithm is very small. Unfortunately, the new algorithm is computationally expensive in spite of the fact that approximately only 100 iterations are required to get a well-pleasing estimates. It is hoped that a parallel implementation of GA will help to alleviate this problem.

6. REFERENCES

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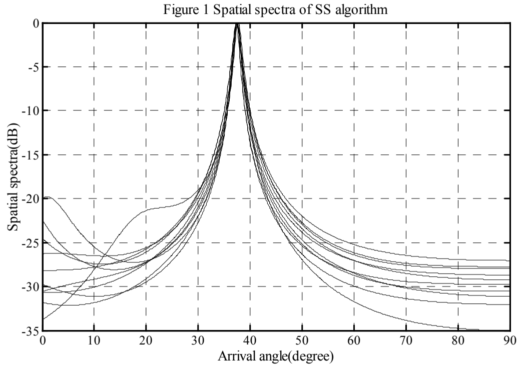

Figure 1 Spatial spectra of SS algorithm

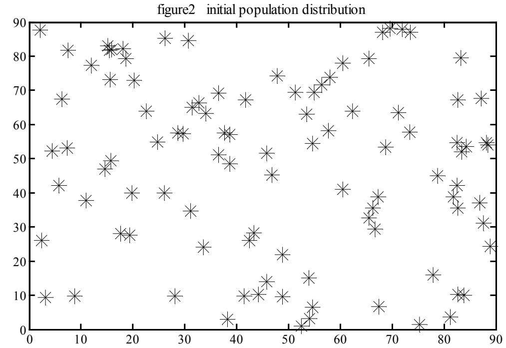
figure2 initial population distribution

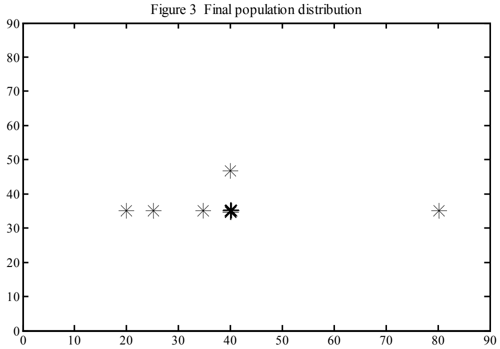
Figure 3 Final population distribution

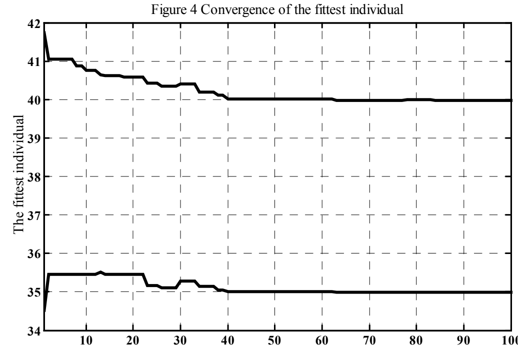

Figure 4 Convergence of the fittest individual