Reactance-Domain MUSIC DOA Estimation Using Calibrated Equivalent Weight Matrix of ESPAR Antenna

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1 Introduction

The Electronically Steerable Parasitic Array Radiator (ESPAR) antenna[1] has been proposed as an adaptive circular array for wireless user terminals. It has recently been reported that high resolution DOA (Direction-of-Arrival) estimation such as the MUSIC algorithm[2] can be applied to ESPAR antenna as a reactance-domain MUSIC algorithm, although it has only a single-port output. Some simulations show that an equally sharp MUSIC spectrum can be obtained as that obtained with a conventional array antenna[3]. An experimental result also presents an actual DOA estimation towards one signal source successfully in an anechoic chamber[4]. However, calibration of the antenna lacks sufficiency because the MUSIC spectrum may not have a sharp peak. In this paper, we propose a calibration method of the ESPAR antenna by calculating an "equivalent weight matrix", which is a matrix composed of several equivalent weight vectors forming directional radiation patterns, and present an experiment result that achieves a sharp MUSIC spectrum in an anechoic chamber.

2 Reactance-domain MUSIC algorithm

The reactance-domain MUSIC algorithm is a high resolution DOA estimation using reactance-domain signal processing of the ESPAR antenna. The reactance-domain signal processing creates a correlation matrix of several signal sequences received by revolving a directional radiation pattern. The MUSIC algorithm is then applied to the obtained correlation matrix. The ESPAR antenna is able to form several directional patterns by changing the reactance values of varactors circularly as shown in Tab. 1. The "*max*" and "*min*" mean maximum and minimum values of varactors loaded in parasitic elements around, respectively.

Tab. 1 Varactor sets [unit:Ω] forming omni and directional patterns

m	pattern	x_{m1}	x_{m2}	x_{m3}	x_{m4}	x_{m5}	x_{m6}
0	omni	*max*	*max*	*max*	*max*	*max*	*max*
1	0 ° directional	***min***	*max*	*max*	*max*	*max*	*max*
2	60 ° directional	*max*	***min***	*max*	*max*	*max*	*max*
3	120 ° directional	*max*	*max*	***min***	*max*	*max*	*max*
4	180 ° directional	*max*	*max*	*max*	***min***	*max*	*max*
5	240 ° directional	*max*	*max*	*max*	*max*	***min***	*max*
6	300 ° directional	*max*	*max*	*max*	*max*	*max*	***min***

When we assume that only a wave is arriving at the ESPAR antenna, the received signal of the m-th directional pattern ($m = 0,1,\cdots, 6$) is given by

$$y_m(t) = \mathbf{w}_m{}^T \mathbf{a}(\phi_a)u(t)+n(t) \quad , \tag{1}$$

where $\mathbf{a}(\phi_a)$ and $u(t)$ are a steering vector with arrival angle ϕ_a and an arrival signal at the center element, $n(t)$ is a thermal noise component and ^{T} denotes transpose. $\mathbf{w}_m$ is an equivalent weight vector of the ESPAR antenna and is expressed by

$$\mathbf{w}_m = 2z_s(\mathbf{Z} + diag[z_s \ jx_{m1} \cdots jx_{m6}])^{-1}\mathbf{u}_0 \tag{2}$$

using an impedance matrix $\mathbf{Z}$ with mutual coupling among elements. z_s is an internal impedance of a receiver. $\mathbf{u}_0$ is a unit vector expressed by $[1,0,0,0,0,0,0]^T$.

Here, when we define an equivalent weight matrix "$\mathbf{W}$" as a matrix composed of seven equivalent weight vectors "$\mathbf{W} = [\mathbf{w}_0 \ \mathbf{w}_1 \ \cdots \ \mathbf{w}_6]^T$ " and express the received signal sequence "$\mathbf{y}(t)$" as $\mathbf{y}(t) = [y_0(t) \ y_1(t) \ \cdots \ y_6(t)]^T$, the $\mathbf{y}(t)$ and the correlation matrix $\mathbf{R}_{yy}$ are given by

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$$\mathbf{y}(t) = \mathbf{W}\ \mathbf{a}(\phi_a)u(t)+\mathbf{n}(t) \tag{3}$$

and

$$\mathbf{R}_{yy} = E[\mathbf{y}(t)\mathbf{y}^H(t)] = \sum_{l=1}^{L} \lambda_l\, \mathbf{e}_l \mathbf{e}_l^H \quad , \tag{4}$$

respectively, where $\mathbf{n}(t)$ is a noise vector and ^{H} denotes helmitian transpose. $E[\cdot]$ stands for an expectation operation. Let L be the total number of varactor sets and L = 7 in the case of Tab. 1. λ and $\mathbf{e}$ are an eigenvalue and an eigenvector, respectively. In the reactance-domain MUSIC algorithm, each pattern needs to receive the same transmitted signal sequence. So the transmitter sends the signal sequence seven times periodically, as shown in Fig. 1 and Fig. 2.

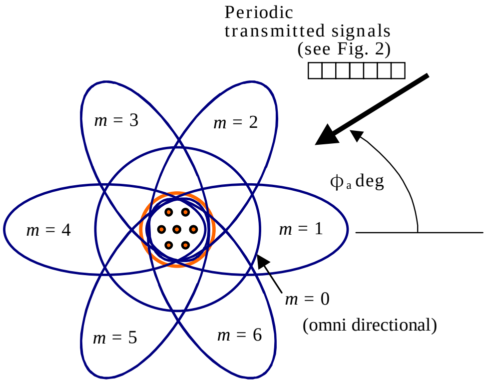

Fig. 1 Revolving a directional pattern

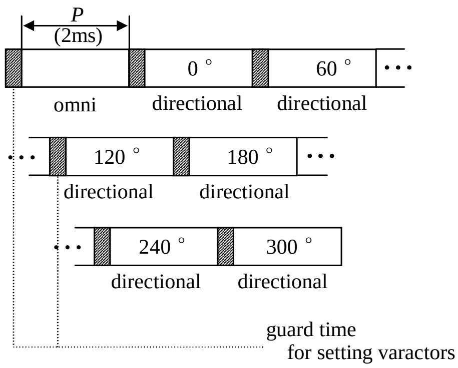

Fig. 2 Periodic transmitted signals

Now, when we express a noise-subspace composed of noise eigenvectors as $\mathbf{E}_N \equiv [\mathbf{e}_2\ \mathbf{e}_3\ \cdots \mathbf{e}_L]$, the MUSIC spectrum is computed by the following equation.

$$P_{MUSIC}(\phi) = \frac{\mathbf{a}^H(\phi)\mathbf{W}^H\mathbf{W}\mathbf{a}(\phi)}{\mathbf{a}^H(\phi)\mathbf{W}^H\mathbf{E}_N\mathbf{E}_N^H\mathbf{W}\mathbf{a}(\phi)} \tag{5}$$

3 Calibration employing signal subspace approach

Each component of the impedance matrix $\mathbf{Z}$ in Eq. (2) has unknown parameters. Though the $\mathbf{Z}$ can be calculated by the moment method, and so on, we have not yet found an accurate $\mathbf{Z}$. Therefore, the equivalent weight matrix $\mathbf{W}$ including $\mathbf{Z}$ has to be calibrated in order to obtain a sharp MUSIC spectrum. This section describes a way to calibrate the ESPAR antenna using a signal eigenvector corresponding to a signal eigenvalue.

At first, from Eq. (3) and Eq. (4), the following equations can be obtained:

$$\mathbf{R}_{yy} = P\mathbf{W}\mathbf{a}(\phi_a)\mathbf{a}^H(\phi_a)\mathbf{W}^H + \sigma^2\mathbf{I} \quad , \tag{6}$$

$$\mathbf{R}_{yy} = \lambda_1\mathbf{e}_1\mathbf{e}_1^H + \mathbf{E}_N\mathbf{\Lambda}_N\mathbf{E}_N^H \quad . \tag{7}$$

By the comparison of Eq. (6) and Eq. (7), we obtain a propotional relationship: $\mathbf{W}\mathbf{a}(\phi_a) \propto \mathbf{e}_1$. (8)

In these equations, P is an expectation of the arrival signal power "$P = E[|u(t)|^2]$". σ^2 and $\mathbf{I}$ are a thermal noise power and an identical matrix, respectively. Λ_N means a diagonal matrix with six eigenvalues corresponding to the thermal noise power σ^2, "$\Lambda_N = diag[\lambda_2\ \lambda_3 \cdots \lambda_L]$". λ_1 and $\mathbf{e}_1$ are an eigenvalue and an eigenvector forming signal-subspace, respectively.

Next, we calculate the signal-eigenvector $\mathbf{e}_1^{(n)}$ using a received signal sequence from the n-th direction (n = 1, $\cdots$, N) to derive the following equation:

$$[\mathbf{e}_1^{(1)}\ \mathbf{e}_1^{(2)} \cdots \mathbf{e}_1^{(N)}] = \mathbf{W}[\mathbf{a}(\phi_1)\,\mathbf{a}(\phi_2)\cdots\mathbf{a}(\phi_N)] \,. \tag{9}$$

Finally, after we replace the two parts of Eq. (9), $[\mathbf{e}_1^{(1)}\ \mathbf{e}_1^{(2)} \cdots \mathbf{e}_1^{(N)}]$ and $[\mathbf{a}(\phi_1)\ \mathbf{a}(\phi_2)\ \mathbf{a}(\phi_N)]$, with the two matrices $\mathbf{E}_s$ and $\mathbf{A}$, respectively, the $\mathbf{W}$ can be calculated by $\mathbf{W} = \mathbf{E}_s\mathbf{A}^H(\mathbf{A}\mathbf{A}^H)^{-1}$. (10)

Concept of this equation scheme will also be found in [5].

4 Experiment and its results

A measurement system in an anechoic chamber is illustrated in Fig. 3. Measurement conditions are shown in Tab. 2. The transmitter with a horn antenna sends periodic signal sequences seven times, and the ES-PAR antenna receives the signals by seven directional patterns. Carrier synchronization and symbol synchronization between the transmitter and the receiver correspond completely due to clock reference through wired cables. In the anechoic chamber, the distance between the transmitter and the receiver is 18 m and each height is about 5 m high. The received data for angle estimation is different from the data for calibration.

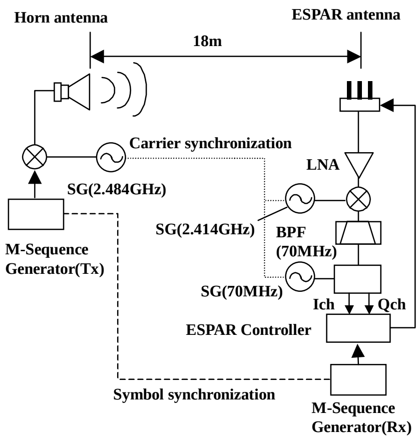

Fig. 3 Measurement system

Tab. 2 Experiment conditions

items		condition
Frequency		2.484GHz
Input SNR		20dB
Polarization		Vertical
Modulation		BPSK
Varactors in Tab. 1	*max*	-4.77 Ω
	min	-90 Ω
Signal sequence		P = 1000 symbol
Number of DOAs for calibration		N = 12 on every 30deg
DOAs for calibration		0, 30, 60, ⋯, 300, 330 deg
DOAs for angle estimation		0, 20, 50, 80, 110, 140, 170 deg

An experiment result of the MUSIC spectrum with the proposed calibration is shown in Fig. 4 with comparison to the case of no calibration. The calibration provides a sharp spectrum and accurate estimation angle. Each estimation angle and spectrum peak on the seven DOA cases is summarized in Tab. 3. It is clear that calibration reduces the estimation error to a few degrees at most.

"Without calibration" means that the calculated **W** is employed in Eq. (5). **W** is not calibrated in the experiment. In this case, each row vector in the equivalent weight matrix **W** is calculated by using Eq. (2). The impedance matrix **Z** in Eq. (2) is computed by a simulation model using the moment method. Therefore, the **Z** may be a little different from the actual impedance matrix. Thus, in the case of no calibration in Fig. 4, the MUSIC spectrum becomes quite dull. The calibration method to modify this difference is required.

For the MUSIC spectrum with calibration in Fig. 4, a lower spectrum peak is observed in the case of DOA = 170 deg. This may be because the varactor sets in Tab. 1 are not so suitable. In order to investigate how much the sets of varactors influence the calibration of the ESPAR antenna, we attempt to measure the MUSIC spectrum by using 6 directional patterns except the omni pattern, as shown in Tab. 4, for example. For comparison, the MUSIC spectrum using 7 patterns and the MUSIC spectrum using 6 patterns are exhibited in Fig. 5. The received data is quite similar in both cases. The equivalent weight matrix **W**s are calibrated independently in each case by Eqs. (4), (9) and (10). In the case using 6 patterns, L in Eq. (4) is equal to 6.

In the case using 6 directional patterns except the omni pattern, the MUSIC spectrum cannot be formed. In DOA = 20, 50, and 80 deg, though spectrum peaks are observed near actual DOAs, the peaks are much lower than those when using 7 patterns. Therefore, it is found that using 7 patterns is more suitable than using 6 patterns. We conclude that the accuracy of DOA estimation depends on sets of varactors forming several directional patterns when the calibration method described in section 3 is applied to the reactance-domain MUSIC algorithm of the ESPAR antenna. We need to consider more suitable sets of varactors in the future.

5 Conclusion

This paper present a calibration method of the equivalent weight matrix for the reactance-domain MUSIC algorithm of the ESPAR antenna. This is a way of calibrating an ESPAR antenna by employing a signal eigenvector corresponding to a signal eigenvalue. We adopt seven directional patterns including an omni pattern to obtain a correlation matrix. In experiments, we can find that the proposed calibration method provides a higher spectrum peak of about 20 dB and an accurate estimation angle. We conclude that accurate DOA estimation depends on sets of varactors forming several directional patterns.

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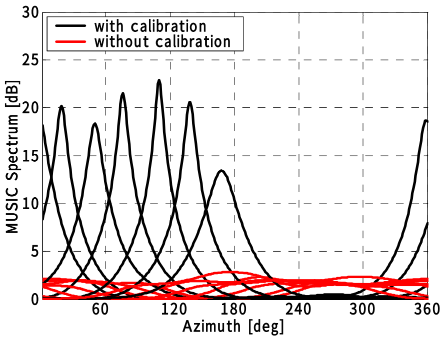

Fig. 4 An experiment result of MUSIC spectrum with calibration. DOAs = 0, 20, 50, 80, 110, 140 and 170 deg.

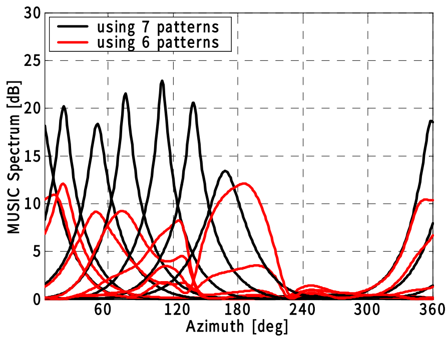

Fig. 5 MUSIC spectrums in cases using 7 patterns and 6 patterns. DOAs = 0, 20, 50, 80, 110, 140 and 170 deg.

Tab. 3 Estimation angles and spectrum peaks of 7 DOAs.

DOA [deg]		0	20	50	80	110	140	170
without calibration	angle [deg]	6	10	247	123	131	72	6
	peak [dB]	2.14	1.96	2.35	2.30	1.97	1.99	2.85
with calibration	angle [deg]	-1	-1	0	-4	0	-1	-2
	peak [dB]	18.7	20.2	18.4	21.6	22.9	20.6	13.4

Tab. 4 Varactor sets [unit:Ω] forming directional patterns

m	pattern	x_{m1}	x_{m2}	x_{m3}	x_{m4}	x_{m5}	x_{m6}
1	0 ° directional	**-90**	-4.77	-4.77	-4.77	-4.77	-4.77
2	60 ° directional	-4.77	**-90**	-4.77	-4.77	-4.77	-4.77
3	120 ° directional	-4.77	-4.77	**-90**	-4.77	-4.77	-4.77
4	180 ° directional	-4.77	-4.77	-4.77	**-90**	-4.77	-4.77
5	240 ° directional	-4.77	-4.77	-4.77	-4.77	**-90**	-4.77
6	300 ° directional	-4.77	-4.77	-4.77	-4.77	-4.77	**-90**