AUXILIARY POTENTIALS IN PSEUD OCHIRAL OMEGA MEDIA

Rostam Saraei* and J. Rashed-Mohassel
Center of Excellence on Applied Electromagnetic Systems
ECE Dept., Faculty of Engineering, Tehran University, P. O. Box 14395-515 Tehran, IRAN, Fax: +(98-21) 877-8690
saraei@khorshid.ece.ut.ac.ir jrashed@ut.ac.ir

Abstract

The expressions for scalar and vector potentials due to axial electric and magnetic current sources within pseudochiral omega media are derived in this work. The $\vec{E}$ and $\vec{H}$ fields in terms of these potentials are obtained. The method of analyzing the EM fields in spectral domain due to a VED source outside a pseudochiral omega half space is introduced. It is observed that for the case of axial current sources, similar to common dielectric media, there is not magneto-electric cross coupling via current sources in pseudochiral omega media.

Index terms: Pseudochiral, omega media, Auxiliary potentials, anisotropy

1. Introduction

In past decades; much attention has been paid to the electromagnetic properties of complex media. A class of these media is known as pseudochiral omega media. Such new types of complex media which is nonchiral can be realized by embedding two orthogonal sets of small omega shaped metals with planes normal to x- and y- axes in a dielectric host medium [1]. These media have applications as: phase shifters [2], pseudochiral point source antennas [3], and pseudochiral waveguides [4]. Although the constitutive parameters of complex media consisting of omega particles is studied [5], the analysis of EM fields by applying auxiliary potentials is not reported. In the present article, scalar and vector potentials are introduced in order to investigate the problem of axial sources and the corresponding fields. The expressions for these potentials due to localized axial electric and magnetic current sources in unbounded pseudochiral omega medium are obtained. Finally the problem of a VED outside a pseudochiral omega half space was investigated.

2. Scalar and vector potentials due to axial current sources

Consider an unbounded pseudochiral omega media with the constitutive relations;

$$\vec{D}=\overline{\overline{\varepsilon}}\cdot\vec{E}+j\Omega^{*}\overline{\overline{J}}\cdot\vec{H} \qquad , \qquad \vec{B}=\overline{\overline{\mu}}\cdot\vec{H}+j\Omega^{*}\overline{\overline{J}}\cdot\vec{E} \tag{1}$$

Where; $\overline{\overline{\varepsilon}}$ *and* $\overline{\overline{\mu}}$ refer to permittivity and permeability uniaxial tensors with transverse component, (t) and the normal component, (n) respectively.

$$\overline{\overline{\varepsilon}}=\varepsilon_t^{*}\overline{\overline{I}}_t+\varepsilon_n^{*}\hat{z}\hat{z} \quad and \quad \overline{\overline{\mu}}=\mu_t^{*}\overline{\overline{I}}_t+\mu_n^{*}\hat{z}\hat{z} \tag{2}$$

With:

$\varepsilon_t^* = \varepsilon_0 \varepsilon_t$, $\varepsilon_n^* = \varepsilon_0 \varepsilon_n$, $\mu_t^* = \mu_0 \mu_t$, $\mu_n^* = \mu_0 \mu_n$ and $\Omega^* = \sqrt{\varepsilon_0\, \mu_0}\, \Omega$

Where Ω (omega parameter) is dimensionless. $\hat{z}$ denotes the unit vector orthogonal to the plane including the stems of omega particles and $\bar{\bar{I}}_t = \hat{x}\,\hat{x} + \hat{y}\,\hat{y}$ is the transverse unit dyadic with $\bar{\bar{J}} = -\hat{z} \times \bar{\bar{I}}_t$.

Assuming an axial electric current source $\vec{J}_e = J_{ez}\,\hat{z}$, our purpose is to find $\vec{E}$ and $\vec{H}$ fields. We introduce the potentials $\vec{A}_{\Omega z}$ and $\Phi_{e\Omega}$ in pseudochiral omega media and decompose the vectors and vector operators into axial components along the z-axis and transverse components orthogonal
to the z-axis i.e.;

$$\vec{\nabla} = \vec{\nabla}_t + \frac{\partial}{\partial z}\hat{z} \quad , \quad \vec{A} = \vec{A}_t + A_z\,\hat{z}, \quad \text{Where;} \quad \vec{\nabla}_t = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} \quad , \quad \vec{A}_t = A_x\,\hat{x} + A_y\,\hat{y}$$

with these notations, one can obtain:

$$\vec{E}_t = -\vec{\nabla}_t\,\Phi_{e\Omega} \quad , \quad E_z = -\frac{\partial \Phi_{e\Omega}}{\partial z} - j\,\omega\,A_{\Omega z}\,, \quad \vec{E} = \vec{E}_t + E_z\,\hat{z} \tag{3}$$

and

$$\vec{H}_t = \frac{1}{\mu_t^*}(\vec{\nabla}_t \times A_{\Omega z}\,\hat{z} - j\Omega^*\,\hat{z} \times \vec{\nabla}_t\,\Phi_{e\Omega}) \quad , \quad H_z = 0 \quad , \quad \vec{H} = \vec{H}_t + H_z\,\hat{z} \tag{4}$$

Using constitutive relations and some manipulations the following relations are obtained for auxiliary potentials in terms of a scalar function, f;

$$A_{\Omega z} = (1 - \frac{\Omega^{*2}}{\varepsilon_t^*\,\mu_t^*})\,f + \frac{\Omega^*}{\omega\,\varepsilon_t^*\,\mu_t^*}\frac{\partial f}{\partial z} \quad , \quad \Phi_{e\Omega} = j\left[-\frac{\Omega^*}{\varepsilon_t^*\,\mu_t^*}\,f + \frac{1}{\omega\,\varepsilon_t^*\,\mu_t^*}\frac{\partial f}{\partial z}\right] \tag{5}$$

Where; f, satisfies inhomogeneous Helmholtz's equation:

$$\nabla_e^{*2} f + k_e^2\, f = -\mu_t^*\,J_{ez} \tag{6}$$

with:

$$\nabla_e^{*2} \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\varepsilon_n^*}{\varepsilon_t^*}\frac{\partial^2}{\partial z^2} \quad \text{and} \quad k_e = \omega\sqrt{(\,\varepsilon_n^*\,\mu_t^* - \frac{\varepsilon_n^*}{\varepsilon_t^*}\Omega^{*2}\,)} \tag{7}$$

Therefore, for any axial source J_{ez}, we can determine *f*, and the desired potentials, $A_{\Omega z}$ and $\Phi_{e\Omega}$ and therefore $\vec{E}$ and $\vec{H}$ fields from (3) and (4) respectively.

For localized sources, consider a constant axial localized electric current source;

$\vec{J}_e = J_{ez}\delta(x)\delta(y)\delta(z)\hat{z}$. Using $\vec{J}_e$ in (6), one can obtain:

$$f = \frac{\mu_t^* J_{ez}}{4\pi m_1'} \frac{e^{-jk_e r_e'}}{r_e'} \qquad \text{with: } r_e' = \sqrt{x^2 + y^2 + \frac{\varepsilon_t^*}{\varepsilon_n^*} z^2} \quad \text{and } m_1' = \sqrt{\frac{\varepsilon_n^*}{\varepsilon_t^*}} = \sqrt{\frac{\varepsilon_n}{\varepsilon_t}} \tag{8}$$

Substituting (8) in (5), $A_{\Omega z}$, $\Phi_{e\Omega}$ and therefore $\vec{E}$ and $\vec{H}$ fields can be determined.

For axial magnetic current source, $\vec{J}_m = J_{mz}\hat{z}$, we introduce $\vec{F}_{\Omega z}$ and $\Phi_{m\Omega}$ in terms of a scalar function, g. With regard to duality properties of the medium, and applying, $\vec{E} \to -\vec{H}, \vec{H} \to \vec{E}, A \to -F, \Phi_{e\Omega} \to -\Phi_{m\Omega}$, $f \to -g, J_{ez} \to -J_{mz}, \varepsilon \leftrightarrow \mu$, $\Omega^* \to -\Omega^*$ and $m_1' \to m_2'$, one can analyze the problem. An investigation shows that the potentials $\vec{F}_{\Omega z}$ and $\Phi_{m\Omega}$ can be obtained from (5) by transforming, $\Omega^* \to -\Omega^*$ and $f \to -g$ where g satisfies the corresponding inhomogeneous Helmholtz's equation. For an axial localized magnetic source, the solution depends on distance variable, r_m' and k_m, which is different from the case with an electric source.

3. Vertical electric dipole above a pseudochiral omega half space

Consider a vertical electric dipole (VED) $\vec{J}_e = J_{ez}\,\delta(z-h)\hat{z}$ located in air, z>0, while region (2), z<0 is a pseudochiral medium. With regard to section (2), we assume incident and reflected potentials for air and a transmitted potential in pseudochiral medium. Applying a two- dimensional Fourier transform and Sommerfeld identity [6], and imposing the interface conditions at z = 0, one can find two equations in terms of reflection, R_m and transmission, T_m coefficients, which yields;

$$R_m = \frac{\beta_1 L_1 - L_2}{\beta_1 L_1 + L_2} \quad , T_m = \frac{2}{\beta_1 L_1 + L_2} \quad \text{with: } L_2 = \frac{\omega\varepsilon_0}{m_1' \beta_2 \varepsilon_t^*}\left(\frac{\beta_2}{m_1' \omega} + j\Omega^*\right) \text{ and}$$

$$L_1 = \frac{1}{m_1' \beta_2}\left(1 - \frac{\Omega^*}{\varepsilon_t^* \mu_t^*} + \frac{\Omega^{*2}}{\varepsilon_t^* \mu_t^*}\right) + j\,\frac{\Omega^*}{m_1' \omega\, \varepsilon_t^* \mu_t^*}\left(1 - \frac{1}{m_1'}\right). \tag{9}$$

Where; β_1 and β_2 are propagation constants along z direction in the two regions. With (9), and the relations in section (2) in spectral domain, it is possible to obtain the potentials and fields in both regions.

4. Conclusion

The scalar and vector potentials due to axial current sources in pseudochiral omega media, $\Phi_{e\Omega}, \Phi_{m\Omega}, A_{\Omega z}$ and $F_{\Omega z}$ were introduced in this work and $\vec{E}$ and $\vec{H}$ fields were obtained using the corresponding potentials. For the case of magnetic current source, applying the duality transformations for fields and $\Omega^* \to -\Omega^*$, one can find the potentials, $F_{\Omega z}$ and $\Phi_{m\Omega}$, and the EM fields. As it is seen, the potentials $A_{\Omega z}$ and $\Omega_{e\Omega}$

(or $F_{\Omega z}$ and $\Phi_{m\Omega}$) are obtained in terms of a scalar function ,f, (or g). Where; ,f, depends on the electric current source, the distance variable r_e', and the wave number k_e. The distance r_e' depends only on spatial coordinates and permittivity tensor, $\overline{\overline{\varepsilon}}$. The function ,g, is related only to magnetic current source and distance variable, r_m' and the wave number, k_m. The distance variable, r_m' depends only on permeability tensor $\overline{\overline{\mu}}$ and spatial coordinates. Both distance variables r_e' and r_m' are independent of omega parameter, Ω^*, while wave numbers k_e and k_m depend on the omega parameter. It is observed that an axial electric (or magnetic) current source produces $\vec{E}$ and $\vec{H}$ fields with zero axial magnetic (or electric) field component, $H_z = 0$, (or $E_z = 0$). Therefore in the pseudochiral omega media similar to common dielectric media, there is not any cross coupling between electric and magnetic fields via axial current source. This is in contrast to chiral media, in which due to chirality parameter either of electric or magnetic current sources along the z direction produces $\vec{E}$ and $\vec{H}$ fields both with nonzero z components ($E_z \neq 0, H_z \neq 0$) [6]. Generally speaking, in the case of axial electric current sources, the physical behavior of EM fields is properly different from the case with axial magnetic current sources. The problem of a VED outside a pseudochiral omega half space was investigated. Introducing reflection and transmission coefficients for the interface and a spectral domain analysis, one can find the appropriate potentials and therefore the corresponding fields.

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