Evaluation Of Singular And Near-Singular Potential Integrals

Donald R. Wilton, Michael Amin Khayat*
Department of Electrical and Computer Engineering
University of Houston, Houston, TX 77204-4005
wilton@uh.edu, michael.khayat@mail.uh.edu

The integral equations of electromagnetics often involve singular potential integrals that require special numerical considerations for their evaluation. Unbounded (but integrable) singularities usually occur, for example, in the kernels of so-called self-terms in the method of moments whenever testing and source subdomains coincide. Often no numerical quadrature rules exist for directly handling these singularities. In such situations, the *singularity subtraction* or *singularity cancellation methods* are often used. In the *singularity subtraction* approach, terms having the same asymptotic behavior as the integrand at the singularity are first removed from the integrand, resulting in a bounded difference integrand that is integrated numerically. An analytically performed integration of the subtracted singular term is then added back to the numerically integrated terms to complete the potential evaluation. Despite widespread usage, however, the singularity subtraction method has a number of disadvantages: (1) The integrand's higher order derivatives remain unbounded and limit the convergence rate of the difference integral. (2) The analytical integration becomes increasingly involved as the complexity of bases, geometry, and Green's functions increase. (3) Object-oriented coding approaches are difficult to implement since each analytically-evaluated self term integral links the source subdomain, basis function, and the asymptotic form of the Green's function.

To extend the capabilities and accuracy of general-purpose codes, we have recently abandoned the subtraction method in favor of purely numerical quadrature schemes, which we report here. These schemes employ *singularity cancellation methods*. In contrast to the singularity subtraction method, the resulting integrand is analytic in the transformed variables and hence is amenable to integration by a Cartesian product of Gauss-Legendre rules. An example of the singularity cancellation method is the so-called Duffy method [M. G. Duffy, *SIAM J. Num. Anal.*, **19**, pp.1260-1262, 1982]. The Duffy method, however, has two major drawbacks: (1) It leaves an angular dependence about the singular point in the resulting integrand. (2) It does not work well when applied to *nearly-singular* integrals that occur when an observation point is *near* a source point.

In this presentation we present a purely numerical singularity cancellation method that not only removes the angular dependence about the singular point, but also is effective in computing nearly-singular integrals. We present both analysis and numerical results for wire, triangle, quadrilateral, brick, tetrahedral and prism subdomains.