THE DISPERSION CHARACTERISTICS OF PBG WITH COMPLEX MEDIUM BY USING NON-YEE GRID HIGHER ORDER FDTD METHOD*

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1. Introduction

Photonic band-gap (PBG) materials [1] have recently attracted significant interest in the microwave region to suppress the surface wave and to improve components performances [2]. The dispersion characteristics of PBG structure have been analyzed by various computational methods, among them, the finite difference time-domain (FDTD) becomes quite popular. The authors had derived the FDTD formulation for the homogeneous complex (bi-anisotropic) medium with various degenerated cases [3], however they are complicated and difficult to deal in programming and calculation.

A Non-Yee grid higher order FDTD (NY-FDTD) method was introduced by Liu [4]. In contrast to the FDTD with standard Yee's algorithm, the NY-FDTD method formulated electric- and magnetic- fields at the same central point of each grid, rather than from the staggered grids. This scheme presents an important advantage over the Yee's algorithm that can easily formulate the EM fields in arbitrary complex medium.

In this article, the NY-FDTD formulation of PBG structure in anisotropic medium are derived, and then applied to compute the dispersion curves of 2-D PBG structure. The numerical results for isotropic medium are good agreement with that from traditional FDTD method. Then the dispersion curves for anisotropic media are provided respectively. Which show that both the *TM* wave in PBG structure with magnetic- anisotropic medium and the *TE* wave in PBG structure with electric- anisotropic medium, possess a enhanced bandwidth of the first band-gap and also an increment of the number of bandgaps, comparing to that PBG with isotropic medium.

2. Non-Yee Grid Higher Order FDTD

In a Non-Yee grid the sampling point of both the electric- /magnetic- fields are placed at the center of grid; the spatial forward-difference for magnetic fields and backward-difference for electric fields are expressed as:

$$\frac{\partial H_l(i,j)}{\partial x} = \sum_{m=-M-1}^{M} a(m)\cdot H_l(i+m,j), \quad \frac{\partial E_l(i,j)}{\partial x} = \sum_{m=-M}^{M+1} b(m)\cdot E_l(i+m,j), \tag{1}$$

where $l = y$ or z; $b(-m-1) = -a(m)$, $m = -M-1, -M, \ldots, M-1, M$.

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In order to determine the coefficients $\{a(m)\}$, by means of a Taylor series expansion, a matrix equation with *N*-th order accuracy is formulated as:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ M^1 & (M-1)^1 & (M-2)^1 & \ldots & -M^1 & -(M+1)^1 \\ M^2 & (M-1)^2 & (M-2)^2 & \ldots & M^2 & (M+1)^2 \\ M^3 & (M-1)^3 & (M-2)^3 & \ldots & -M^3 & -(M+1)^3 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ M^{N-1} & (M-1)^{N-1} & (M-2)^{N-1} & \ldots & (-M)^{N-1} & (-M-1)^{N-1} \end{bmatrix} \begin{bmatrix} a(M) \\ a(M-1) \\ a(M-2) \\ \ldots \\ a(-M) \\ a(-M-1) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (2)$$

and then be solved for the cases of :

M=1, *N*=2

a (1)	*a* (0)	*a* (-1)	*a* (-2)
0.3333333	0.5000000	-1.0000000	0.1666667

M=2, *N*=3

a (2)	*a* (1)	*a* (0)	*a* (-1)	*a* (-2)	*a* (-3)
-0.0500000	0.5000000	0.3333333	-1.0000000	0.2500000	-0.0333333

M=3, *N*=4

a (3)	*a* (2)	*a* (1)	*a* (0)
0.0095238	-0.1000000	0.6000000	0.2500000
a (-1)	*a* (-2)	*a* (-3)	*a* (-4)
-1.0000000	0.3000000	-0.0666667	0.0071428

M=4, *N*=5

a (4)	*a* (3)	*a* (2)	*a* (1)	*a* (0)
-0.0019841	0.0238095	-0.1428571	0.6666667	0.2000000
a (-1)	*a* (-2)	*a* (-3)	*a* (-4)	*a* (-5)
-1.0000000	0.3333333	-0.0095238	0.0178571	-0.0015873

3. Numerical Stability

By decomposing the Maxwell's equations into temporal and spatial eigenvalue problems, and employing the non-Yee higher order spatial differences, a numerical stability condition is deduced as:

$$\Delta t < \frac{1}{2c} \frac{1}{\sqrt{(\frac{1}{\Delta x})^2 (A_0 + 2\sum_{n=1}^{2M+1} A_n) + (\frac{1}{\Delta y})^2 (A_0 + 2\sum_{n=1}^{2M+1} A_n)}} \quad (3)$$

Where $A_0 = \sum_{i=-M}^{M-1} a(i)^2$, $A_n = \sum_{i=-M}^{M-1-n} a(n) \cdot a(n+i)$

4. Numerical Dispersion Relation

Substituting the fields of monochromatic plane wave into the difference form of Maxwell equations, a numerical dispersion relation is formulated as:

$$\{A_0 + \sum_{n=1}^{2M+1} A_n \cdot [\cos(n \cdot A \cdot \tau) + \cos(n \cdot B \cdot \tau)]\} = 2(\frac{1}{q})^2 \sin^2(\pi \sigma q) \quad (4)$$

Where $A = \pi\sigma\cos\theta,\ B = \pi\sigma\sin\theta$; $\sigma = \Delta s / \lambda$, $\Delta s = \Delta x$ or Δy; λ is wavelength; θ is incident angle; $q = C\Delta t / \Delta s$, $\tau = c / v_p$, v_p is phase velocity, $k = \omega / v_p$.

For given σ, q, θ, and τ corresponding to the phase error = (1-τ)×100%, by comparing the family of numerical dispersion relation curves, one can find that phase error in the 6-th order FDTD method (N = 6) is the smallest. For example, when $\sigma = 1/8,\ 1/10$, the phase error is less than 0.25%.

5. Numerical Results

In order to verify the feasibility and validity of dispersion characteristics of the PBG structure analyzed by using NY-FDTD method, a sample of PBG structure with isotropic medium is calculated at first. Its results are very agreement with that using traditional FDTD method (Fig.2). Then we turn to the calculated result of dispersion characteristics, shown as curves (Fig.3), of the PBG structure with anisotropic medium by using NY-FDTD method.

Case 1: A 2-D dielectric-rods PBG with relative permittivity $\varepsilon_r = 10.2$ and dielectric filling ratio $\beta = 0.18$, the side length of its periodic square cell L=12cm, to calculate the k-$f(\omega)$ Brillouin diagram for TM wave propagation.

Case 2: A 2-D magnetic-anisotropic-rods, PBG with the constitutive relation $\varepsilon_r = 10.2$ and $\vec{\mu} \Rightarrow \mu_0 \begin{pmatrix} \mu_{xx} & \mu_{xy} & 0 \\ \mu_{yx} & \mu_{yy} & 0 \\ 0 & 0 & \mu_z \end{pmatrix}$, filling ratio $\beta = 0.25$, the side length of its periodic square cell L =12 cm, to calculate the k-$f(\omega)$ Brillouin diagram for TM wave propagation under the parameters $\mu_{xx} = \mu_{yy} = 10,\ \mu_z = 1$; and (i) $\mu_{xy} = \mu_{yx} = g$ (magnetic-crystal) or (ii) $\mu_{xy} = jg,\ \mu_{yx} = -jg$ (Ferrite), respectively.

Fig.3 shows that the magnetic-anisotropic rods PBG can increase the width of first band- gap and increace the number of band-gap for TM wave propagation.

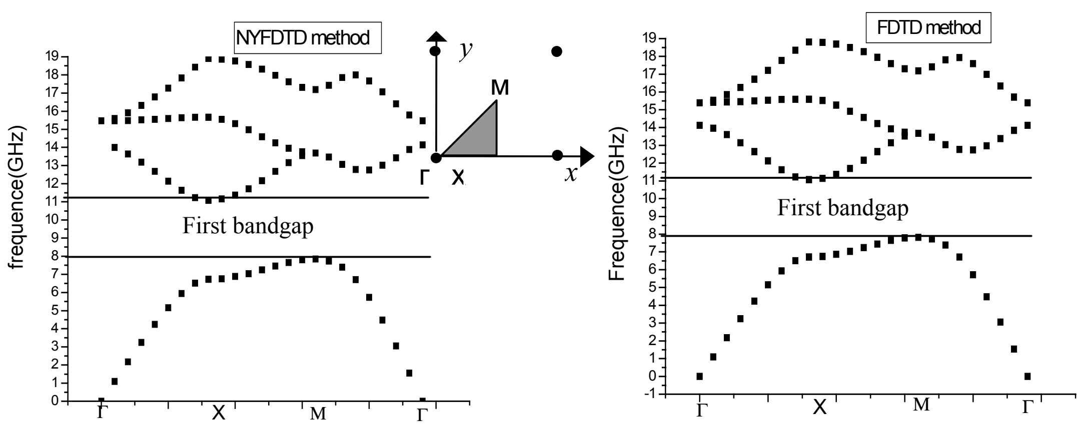
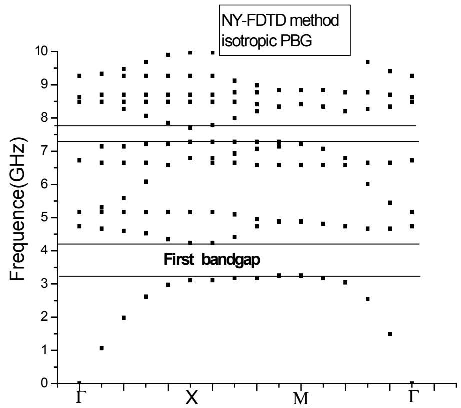
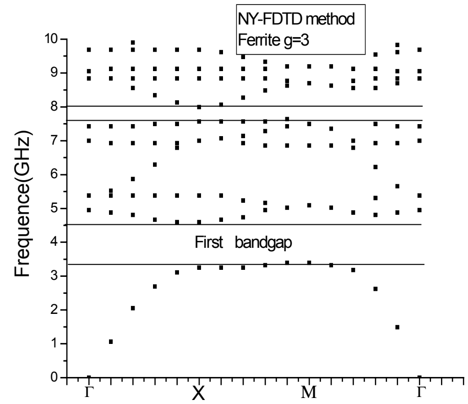
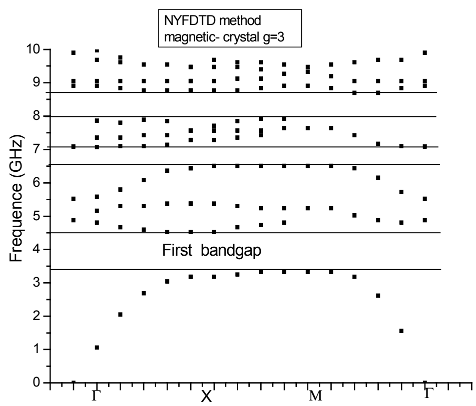


Fig. 2 Comparison between NY-FDTD and FDTD methods







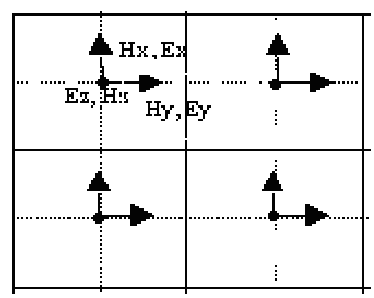


Fig. 3 Comparision between different media by using NY-FDTD method

Fig.1 location of Fields in the non-Yee grid

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