Diacoptic Basis Function Grouping Techniques Applied to the Method of Moment Solution of (M)MIC-Structures

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1. Introduction

For the analysis and design process of (M)MIC structures, electromagnetic simulators based on the Method of Moments (MoM) are widely used due to their high accuracy and good modeling capabilities which have been accomplished over the last years. For the standard MoM using an explicit system matrix, many improvements could be achieved for a faster and more accurate computation of the matrix entries. These improvements are based on space domain evaluations as well as on spectral domain evaluations of the matrix entries leading to a drastical reduction of matrix fill time. Thus, the solution of the linear systems of equations typically remains the most cumbersome part of the MoM.

Here the worst choice in terms of computational effort is the use of standard Gaussian elimination techniques with complete LU decomposition, leading to numerical complexities of $O(N^3)$. Therefore, most contributions dealing with the treatment of MoM matrices favorize iterative solvers, where Krylov subspace methods are mainly in the scope of interest. Conjugate Gradient (CG) methods have been embedded in CG-FFT methods circumventing the evaluation of the entire explicit system matrix. However, a direct application of Krylov subspace methods to MoM matrices typically exhibits a bad convergence behavior, especially with regard to a nonuniform structure discretization [1].

An important class of methods to improve the solution process was initialized by the diacoptic theory of linear antennas. This theory is based on a component decomposition and is therefore not restricted to the analysis of wire antennas but is particularly well suited for (M)MIC structures with its typical circuit component subdivision. Diacoptic strategies are mainly based on the use of so-called Macro Basis functions (MBs), leading to multilevel MoM implementations. In our contribution we first compare different versions of the two-level approach in[3], combined with an advanced description of the macro basis function decomposition. It is shown that the diacoptic entire basis function process in [3] is not the optimal choice due to its high computational complexity and a modified diacoptic procedure with optimized matrix decomposition and a block Jacobi iteration process is proposed.

Furthermore it is shown that the derived matrix decomposition can be used for a very effective preconditioning strategy applied to Krylov subspace methods. It results in excellent convergence properties without the explicit current profile determination of the MBs, which is cumbersome or not reasonable in many cases.

2. Formulation

A typical section of a microwave circuit with components such as patches, junctions and inductors is given in Fig.1. For diacoptic treatment of such structures we have to introduce additional artificial ports, e.g. P1–P7 in Fig. 1. With these ports we subsequently introduce a block subdivision of the structure, leading to 9 blocks B1–B9. Now we define MBs by allocating each port two blocks which are connected by this port, e.g. port 1 connects block 1 and 2, port 3 connects block 3 and 5, etc.. If we excite each port with its allocated blocks in absence of the other blocks using standard MoM, we get in this case a set of 7 MBs with its current profiles. In the next step we formulate an upper level MoM by computing the self and mutual

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couplings of the MBs, leading to a 7×7 matrix. The coupling of e.g. the MBs k and l is computed via

$$Z_{kl} = \sum_{m \in \mathrm{MB}_k} \sum_{n \in \mathrm{MB}_l} I_{mk} Z_{nm} I_{nl} \tag{1}$$

with Z_{nm} the entries of the lower level MoM, $m \in \mathrm{MB}_k$ is the index m of all basis functions allocated to the MB number k, I_{mk} the current profile of the kth MB etc..

Solving for the amplitudes of the MBs and the subsequent superposition of the MB current profiles provides the current distribution of the overall circuit without the need of solving the large system matrix using standard MoM. However, the errors in the current distribution can amount to more than 10 percent compared with the rigouros solution of the standard MoM since the remaining blocks are not considered during the determination of the MB current profiles.

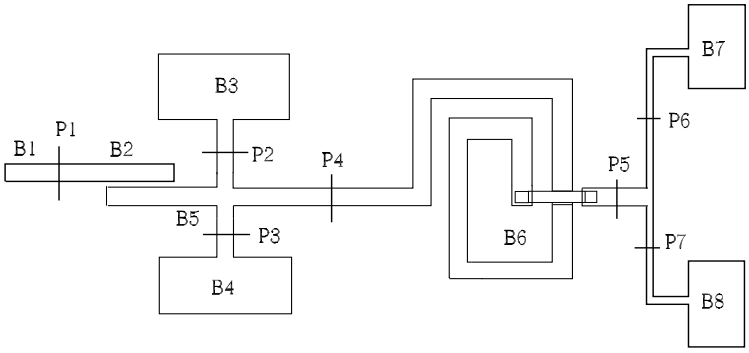

Fig. 1: Typical section of a microstrip circuit with additional ports P1–P7 and allocated blocks B1–B8.

Thus we have to apply a further iterative refinement process. In [3] a block Gauss-Seidel process is applied to each MB. During this process also the remaining blocks are considered which leads to entire domain basis functions called diacoptic functions. Although this method converges very fast in nearly all cases, it exhibits several disadvantages. Since the diacoptic basis functions extend over the whole structure, complete matrix-vector products must be performed within the block Gauss-Seidel process as well as for computing the upper level matrix with eq.(1). This must be done for each MB, leading to a $O(N^3)$ process. Nevertheless, the procedure is about 10–30 times faster than a LU factorization of the whole system matrix, furthermore the storage effort can be drastically reduced since only small parts of the system matrix are needed at the same time.

On the other hand also a block Jacobi method was proposed in [3], based on the process

$$Z^i \cdot I_{n+1} = U - Z^c \cdot I_n \quad , Z = Z^i + Z^c \tag{2}$$

leading to an improved current amplitude vector I_{n+1} starting with an estimated amplitude vector I_n. Z^i is proposed a sparse block diagonal matrix containing the self and mutual couplings of each block forming diagonal submatrices, whereas Z^c comprises all remaining mutual couplings. As starting vector I_0, the current distribution derived by the compressed matrix eq.(1) is used. Unfortunately this process does not converge in most cases. This is due to the fact, that the convergence condition

$\rho(Z^{i^{-1}} Z^c) < 1$

cannot be fulfilled with the chosen matrix decomposition where $\rho(A)$ denotes the spectral radius of a matrix A. This can be traced back to the fact, that Z^c still contains couplings of overlapping basis functions. Thus, the matrix Z^i must be extended in order to comprise all dominant couplings. In our approach, we split off all couplings we have already needed to compute eq.(1).

For the structure in Fig. 1, the first part of the matrix Z^i up to the spiral inductor (block 6) reads:

$$Z^i = \begin{bmatrix} Z_{11} & Z_{12} & 0 & 0 & 0 & 0 \\ Z_{21} & Z_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{33} & 0 & Z_{35} & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} & 0 \\ 0 & 0 & Z_{53} & Z_{54} & Z_{55} & Z_{56} \\ 0 & 0 & 0 & 0 & Z_{65} & Z_{66} \end{bmatrix} \tag{3}$$

Here the Z_{ij} represent submatrices which contain the mutual couplings between block i and block j. For the application of eq.(2) we employ a Cholesky decomposition of Z^i in case of pure microstrip or slot structures (symmetric matrix Z^i). Subsequently for each iteration we have to perform a matrix-vector multiplication $Z^c \cdot I_n$ and a forward/backward substitution to obtain the improved current vector I_{n+1}. The algorithms for the Cholesky decomposition as well as for the forward/backward substitution have been optimized by means of analysing the individual structure of Z^i depending on the circuit topology and block numbering. Thus a further feature detects the grouping of the submatrices by forming upper level submatrices with special treatment of possible matrix overlappings. This allows an adaptive Cholesky decomposition and forward/backward substitution guaranteeing a minimum of computational effort to evaluate eq.(2) for a given matrix Z^i.

Despite of the outlined extension of Z^i compared to a pure block diagonal matrix we get an increasing sparsity of Z^i with an increasing number of blocks, where the diagonal dominance of the resulting matrix Z^i still depends on a proper block numbering scheme. The storage efficiency was further optimized by a sparsification of the matrix Z^i (one-dimensional storage of Z^i with additional index arrays). Possible fill-ins during the Cholesky decomposition are automatically processed by an adaptive array extension and entry updating. By theses measures the storage complexity can drastically reduced down to $O(N)$, but requires the repeated computation of the lowest level couplings Z_{mn} during the matrix-vector multiplications.

Now the matrix Z^i and its Cholesky decomposition is also well-suited as a left and right preconditioner in context with Krylov subspace iterative solvers applied to the equivalent linear system of equations

$C^{-1} Z\, C^{-T} C^T \cdot I = C^{-1} \cdot U$

with C the lower triangular part of the Cholesky decomposition of Z^i. Initially we have also used the approximation of I by the superposition of the MBs using eq.(1) as a starting vector. But further studies have shown that the solution behavior of any Krylov subspace solver is nearly independent from the starting vector even in the case of a starting vector close to the exact solution. Only very few iterations are needed by simply using the normalized excitation vector U as starting vector. This means that we can omit the definition of additional ports and the direct determination of the MB current profiles which is cumbersome in case of ports on wide transmission lines requiring a discretization over the width. Especially in the case of an external wave excitation of the structure a reasonable MB definition seems not possible. Nevertheless we still use a port definition in the input file to describe the connections between the blocks what is still necessary for the composition of the matrix Z^i. Consequently, the macro basis function concept remains inherent in the structure of Z^i.

3. Application

As an application well-suited to illustrate the behavior of the different solution methods with increasing number of unknowns, we have chosen a quadratic slot antenna array with coplanar feeding (1034 unknowns with 6 elements, see Fig. 2 a)) on a silicon halfspace. On the right the required CPU-time for the different solvers dependend on the number of array elements are given (1000 MHz AMD Athlon PC). Each element is subdivided into two blocks (upper and lower half) thus each element just represents one MB leading to a matrix Z^i with a homogeneous submatrix structure. The diacoptic strategy according to [3] (two iterations per MB, less than 1 per cent error in input impedance) already exhibits a drastical improvement against

a Gauss elimination whereas the block-Jacobi process (two additional iterations in eq.(2)) even shows a much better numerical complexity. This is illustrated in detail in Fig. 2 c) demonstrating that Krylov subspace methods with preconditioner based on Z^i (here a Transpose Free Quasi Minimim Residual (TFQMR) method [4]) show the best performance and accuracy (input impedance error $<<$ 1 per cent) with 2–4 iterations. Implementations with a Conjugate Residual (CR) method or even a standard Conjugate Gradient method show only a slightly decreased performance.

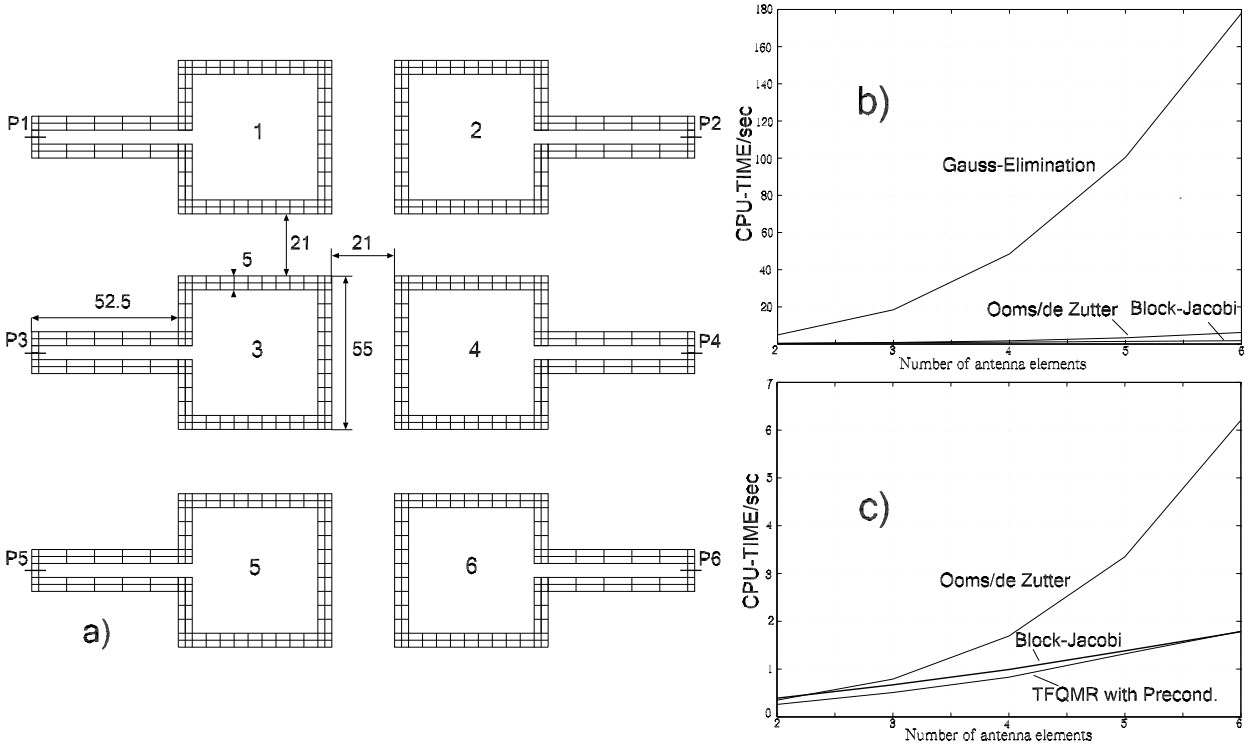

Fig. 2: a) Quadratic slot antenna array with coplanar feeding on silicon halfspace (dimensions in μm), b), c) required CPU-time with different linear equation solving methods

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