A Fast Technique for the Analysis of Infinite Frequency Selective Surfaces

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I. Introduction

Over the past few decades, a number of analytical and numerical techniques have been developed for analyzing the infinite double periodic Frequency Selective Surfaces (FSSs). The techniques for analyzing multi-screen FSS composites can be divided into two general categories: those analyze the entire system simultaneously vs. those that employ the cascading approach based on the Generalized Scattering Matrix (GSM) technique. The former group of methods typically employ the moment method technique [1,2] to determine the unknown current distributions on the FSS screens in one step, albeit at the expense of increasing the number of unknowns approximately N-fold when there are N screens in the composite. The conventional cascading approach, mentioned above, provides a way to overcome these difficulties by employing the GSM technique [3]. In this method, the reflection and transmission properties of FSS screens are first described in terms of scattering matrices, and a linear system approach is subsequently applied to the cascaded screens to derive the scattering parameters for the composite. However, the size of the scattering matrices can become quite large for closely spaced FSSs, and an efficient iterative approach to overcome this problem has recently been introduced by Prakash and Mittra [4]. In all of these approaches, the unit cell of the FSS screen is subdivided into uniform cells by using roof-top basis functions to represent the induced current density on the metallic portions, which is solved by using Electric Field Integral Equation (EFIE). Quite often, a large portion of the unit cell would be occupied by metal, which in turn increases the number of unknowns significantly. An alternative to this problem is to use Magnetic Field Integral Equation (MFIE) and solve for the magnetic current density at the aperture portion, which requires reformulating the original problem. In this paper, we present a fast and efficient technique for analyzing infinite doubly periodic Frequency Selective Surfaces (FSSs) using the equivalence principle by solving the EFIE alone. The numerical efficiency of the proposed approach is illustrated with test cases comprising of multiple FSSs.

II. Theory

The scattering characteristics of infinite double periodic FSS structures are normally evaluated under transverse electric (TE) and/or transverse magnetic (TM) plane wave incidence. In the conventional approach, the unitcell is subdivided into M x N uniform cells, and the unknown current distribution on the metallic portions is expanded into roof-top basis functions. An electric field integral equation (EFIE) is formulated by enforcing the boundary conditions on the metallic portions, which is then solved to obtain the current expansion coefficients [1-4]. However, in many practical situations, a significant portion of the unitcell constitutes of metal, leading to a large number of unknowns. This in turn slows down the convergence of the iterative solvers used for solving the MoM matrix equation [1]. In such a situation, it is desirable to reduce the CPU time as the EFIE has to be solved repeatedly for a wide range of frequency, and the incident angles.

We propose an approach to address the above mentioned problem by using equivalence principle. Here, we form a dual FSS screen by interchanging the metallic and free space regions of the original geometry as shown in Fig.1 in order to reduce the number of unknowns. We first solve the dual screen geometry and obtain its generalized scattering matrix. Next, by employing the equivalence principle, we generate the GSM of the original problem from that of the dual screen geometry. The GSM for the original and the dual screens is identified as 'a' and 'b', respectively. Let the generalized scattering matrix of the dual screen is given by

$$\begin{bmatrix} \mathbf{S}_{b} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \mathbf{S}_{b,11} \end{bmatrix} \begin{bmatrix} \mathbf{S}_{b,12} \end{bmatrix} \\ \begin{bmatrix} \mathbf{S}_{b,21} \end{bmatrix} \begin{bmatrix} \mathbf{S}_{b,22} \end{bmatrix}$$
(1)

The GSM of the original screen is then generated from that of the dual screen by using equivalence principle.

$$\begin{bmatrix} \mathbf{S}_{a,11} \end{bmatrix} = \begin{bmatrix} -\mathbf{S}_{b,21}^{\mathrm{TM}_{i},\mathrm{TM}_{s}} & \mathbf{S}_{b,21}^{\mathrm{TE}_{i},\mathrm{TM}_{s}} \\ \mathbf{S}_{b,21}^{\mathrm{TM}_{i},\mathrm{TE}_{s}} & -\mathbf{S}_{b,21}^{\mathrm{TE}_{i},\mathrm{TE}_{s}} \end{bmatrix}; \begin{bmatrix} \mathbf{S}_{a,12} \end{bmatrix} = \begin{bmatrix} -\mathbf{S}_{b,22}^{\mathrm{TM}_{i},\mathrm{TM}_{s}} & \mathbf{S}_{b,22}^{\mathrm{TM}_{i},\mathrm{TE}_{s}} \\ \mathbf{S}_{b,22}^{\mathrm{TE}_{i},\mathrm{TM}_{s}} & -\mathbf{S}_{b,22}^{\mathrm{TE}_{i},\mathrm{TE}_{s}} \end{bmatrix}$$
(2)

$$\begin{bmatrix} \mathbf{S}_{a,21} \end{bmatrix} = \begin{bmatrix} -\mathbf{S}_{b,11}^{\mathrm{TM}_{i},\mathrm{TM}_{s}} & \mathbf{S}_{b,11}^{\mathrm{TE}_{i},\mathrm{TM}_{s}} \\ \mathbf{S}_{b,11}^{\mathrm{TM}_{i},\mathrm{TE}_{s}} & -\mathbf{S}_{b,11}^{\mathrm{TE}_{i},\mathrm{TE}_{s}} \end{bmatrix}; \begin{bmatrix} \mathbf{S}_{a,12} \end{bmatrix} = \begin{bmatrix} -\mathbf{S}_{b,12}^{\mathrm{TM}_{i},\mathrm{TM}_{s}} & \mathbf{S}_{b,12}^{\mathrm{TM}_{i},\mathrm{TE}_{s}} \\ \mathbf{S}_{b,12}^{\mathrm{TE}_{i},\mathrm{TM}_{s}} & -\mathbf{S}_{b,12}^{\mathrm{TE}_{i},\mathrm{TE}_{s}} \end{bmatrix}$$
(3)

where, the first and second superscript's for each of the matrix entries denote the incident and scattered wave polarization's, respectively. The above procedure is repeated for each of the FSS screens, and the individual GSM's are then cascaded to obtain the overall scattering matrix.

III. Results

The theory presented in the previous section has been used to analyze a FSS structure comprising of two screens. Each of the screens consists of a double square loop shown in Fig.1a, and the two screens are spaced 1cm apart from each other. A 64 x 64 uniform discretization of the unit cell has been used to be able to represent the geometry accurately. For this case, the original problem shown in Fig.1a. has 6988 unknowns, while the one with dual screen shown in Fig.1b. involved just 860 unknowns. The FSS screen has been analyzed under plane wave incidence at $\theta_i = \phi_i = 1^0$, and its GSM is obtained. Next, the GSM of the original FSS is generated according to (2) and (3). In the present example, both the screens are identical and hence have the same GSM. The overall scattering matrix is then obtained by cascading the individual GSM's of each of the FSS screens. The reflection and transmission coefficients of the original structure (Fig.1a) are presented in Fig.3 and 4, respectively. Also, the results obtained by directly

analyzing the original structure by using the CG-FFT are also shown in the same figures for the sake of comparison. Close agreement is observed between the results of the present approach and that of the CG-FFT. The CPU time for the present approach for each frequency point is 30s, while that of the CG-FFT is 133s, on an IBM SP6000 machine.

IV. Conclusion

In this paper, a computationally efficient approach using equivalence principle for the method of moments solution of the infinite doubly periodic frequency selective surfaces has been introduced. This approach involves forming a dual screen geometry with reduced metallization, which in turn leads to a reduction in the number of unknowns. Reformulation of the problem by using MFIE is not necessary, and the existing legacy codes employing the EFIE can easily be modified to accommodate the present approach. Numerical experiments reveal the accuracy and the computational advantage of this technique when compared to the conventional techniques.

References:

- [1] Raj Mittra, C.H. Chan, and T. Cwik, "Techniques for analyzing frequency selective surfaces-A review," Proc. IEEE, vol.76, no.12, pp.1593-1615, Dec. 1988.
- [2] C.H. Chan and Raj Mittra, "On the analysis of frequency selective surfaces using sub domain basis functions," IEEE Trans. Antennas Propagation, vol.38, no.1, pp.40-50, Jan. 1990.
- [3] J.D. Vacchione and Raj Mittra, "Cascade analysis of power distribution in a multilayered dielectric/frequency selective surface system," IEEE AP-S Int. Symposium, San Jose, Vol II, pp.1080-1083, June 1989.
- [4] V.V.S.Prakash and Raj Mittra, "An efficient technique for analysing multiple frequency selective surface screens with dissimilar periods," Microwave Optical Technology Letters, pp.23-27, Oct. 2002.



Fig.1. Unitcell of double square loop FSS screen. (a) original screen, and (b) dual screen. The shaded area represents metallic portion.



Fig.2. Reflection coefficient of the cascaded FSS structure computed using the original screen and its dual.



Fig.3. Transmission coefficient of the cascaded FSS structure computed using the original screen and its dual.