Differential Equation Model for a Through Hole Via in Multilayered Microstrip Circuits

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I. INTRODUCTION

The analysis for a through hole via, which is used as a vertical interconnect line in multilayered microstrip circuits, is a challenge 3-D problem. At sufficiently lower frequency, the size of a via is far smaller than the wavelength of system operating frequency and discontinuity characteristics such as coupling and radiation effects rising from via can be negligible. On basis of quasi-static integral-equation formulations, T. Wang [1] and P. Kok [2] presented a simple lumped circuit element model for the through hole via in a ground plane with inductance and capacitance respectively. Meanwhile, empirical formulas to approximately model via hole were given [3-4]. However, when the operating frequency increases up to a few gigahertz, coupling and radiation effects become critical. In order to properly model the propagation characteristics of the through hole via in the high frequency, enormous efforts have been taken in the past years. More accurate modeling for various via have been published recently by using numerical techniques such as FDTD [5], MoM [6-7], 3D-Mode Matching [8]. However, even for such a simple via hole problem, any 3-D numerical technique needs a lot of computer memory and is too time consuming. In this paper, we present a simple 1-D differential equation, or say a generalized transmission line equation, to model through hole via. The difference between the generalized transmission line equation and the well-known Telegraph's differential equation is that the former can be used in both TEM mode and non-TEM mode problems but the later is only for the TEM mode problem. Furthermore, results for a broad frequency band obtained by the generalized transmission line equation can be agreed with those obtained by Maxwell equations.

III. DIFFERENTIAL EQUATION MODEL

For a 3-D problem like the trough hole via, traditional voltage in the Telegraph's equation can not been defined uniquely, so the Telegraph's equation fails to be used in this kind of problems. However, if we use the generalized transmission line equation, the problem of via hole can be modeled simply. The reason is that the voltage used in the generalized equation is scale potential that can be determined uniquely and the generalized equation has already considered discontinuity effects of both reflection and radiation. The lossless generalized transmission line equation [9] can be rewritten as follows

$$\begin{cases} \dfrac{dV}{dl} = -jwL(l)I(l) + a(l)V(l) \\ \dfrac{dI}{dl} = -jwC(l)V(l) + b(l)I(l) \end{cases} \tag{1}$$

where $a(l)$ and $b(l)$ are local radiation coefficients with energy emission and absorption generated by non-uniform interconnect structures. $V(l)$ and $I(l)$ are the scalar potential and current distributions on the line. $L(l)$ and $C(l)$ are the per-unit-length series inductance and shunt

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capacitance respectively. This generalized equation has been successfully applied to discontinuities such as bends, steps, and low-pass filters [9-10]. In order to use the generalized equation for through vole via, the first thing we need to do is to determine the equation coefficients $a(l)$, $b(l)$, $L(l)$ and $C(l)$, or say to extract parameters of the transmission line at a given operating frequency. To do this, two pairs of potentials and currents on the through hole via are required by means of commercial EM simulation tools such as Zeland IE3D, Ensemble, and Sonet. Usually solutions for both open and short loadings can be used for this purpose. These coefficients have been found that they are slowly changeable with the operating frequency. Usually, the differential equation determined at the given frequency can be used to model the through hole via at a wide frequency range. Once the differential equation model is established, the speed to solve such a 1-D differential equation at different loads and frequencies is much faster than to solve the same problem by using other 3-D numerical techniques.

III. NUMERICAL RESULTS

Fig.1 shows the structure of investigated trough hole via, which consists of two circular lips and a hollow cylinder. The two lips are connected to finite length microstrip lines sided on substrate layers respectively, while the hollow cylinder penetrates through a hole in a ground plane, which sandwiched between two dielectric substrates with the same thickness t = 30mil and the same relative dielectric constant e_r = 9.8. The inner radius of the circular lips is 2mil and the external radius 3.5mil. The metal of microstrip and via is supposed to be lossless and infinitely thin, hence the radius of hollow cylinder has the same size as the inner radius of the circular lips. Since the wire through hole via is used to connect the signal lines, the radius of hole in the ground is much larger than that of hollow cylinder. To determine the differential equation coefficients $a(l)$, $b(l)$, $L(l)$ and $C(l)$ for through hole via, we use full-wave simulation tool, Zeland IE3D to find two independent pairs of on-line potentials and currents at operating frequency 8GHz for open and short loads, respectively. Substituting the two pairs of solutions of potentials and currents into equation (1), then equation coefficients $a(l)$, $b(l)$, $L(l)$ and $C(l)$ can be determined. The determined equation can be used for solving problems with other loads and other frequencies. The comparison of current distributions of the equation for a load with 20Ω with the MoM is given in Fig.2. It shows that they agree very well. From voltages and currents at ports, we can calculate the scattering parameters for the through hole via by transforming impedance matrix. The scattering parameters from 6GHz to 10GHz shows that results obtained by the equation established only at 8GHz are very close to those obtained by the MoM, as shown in Fig.3. For the computation speed of scattering parameters for the above bandwidth of interest, it takes only a few seconds for the differential equation while it needs ten minutes for the MoM.

IV. CONCLUSIONS

A simple distribution circuit has been presented for modeling non-TEM propagation characteristic of the through hole via for the first time. This model has given an excellent result, which is very close to the full-wave MoM simulation within a wide frequency bandwidth, while the speed of the differential equation is much faster than the MoM.

REFERENCES

[1] T. Wang, R. F. Harrington, and J. R. Mautz, "Quasi-static analysis of a microstrip via through a hole in a ground plane," IEEE Trans. Microwave Theory Tech., vol. 36, pp. 1009-1013, June, 1988.

[2] P. Kok, and D. D. Zutter, "Capacitance of a circular sysmmetric model of a via hole including finite ground plane thickness," IEEE Trans. Microwave Theory Tech., vol. 39, pp. 1229-1234, July 1991.

[3] M. E. Goldfarb and R. A. Pucel, "Modeling via hole grounds in microstrip," IEEE Microwave Guided Wave Lett., vol. 1, pp. 135-137, June 1991.

[4] A. W. Mathis and C. M. Butler, "Capacitance of circularly symmetric via for microstrip transmission line," IEE Proceeding of Microwave Antennas Propagation, vol. 141, no.4, pp.265-271, Aug. 1994.

[5] Shuji Maeda, Tatsuya Kashiwa, and Ichiro Fukai, "Full wave analysis of propagation characteristics of through hole using the finite-difference time-domain method," IEEE Trans. Microwave Theory Tech., vol. 39, no. 12, pp. 2154-2159, Dec. 1991.

[6] Qizheng Gu, Y. Eric Yang, and M. Ali Tassoudji, "Modeling and analysis of vias in multilayered integrated circuits," IEEE Trans. Microwave Theory Tech., vol. 41, no. 2, pp. 206-214, Feb. 1993.

[7] Show-Gwo Hsu and Ruey-Beei Wu, "Full wave characterization of a through hole via using the matrix-penciled moment method," IEEE Trans. Microwave Tech., vol. 42, no. 8, pp. 1540-1547, Aug. 1994.

[8] Roberto Sorrentino, Ferdinando Alessandri, Mauro Mongiardo, Gianfranco Avitable, and Luca Roselli, "Full-wave modeling of via hole grounds in microstrip by three-dimensional mode matching technique," IEEE Trans. Microwave Tech., vol. 40, no. 12, pp. 2228-2234, Dec. 1992.

[9] Y. W. Liu, J. S. Hong, W. M. Liu, and K. K. Mei, "Radiation parameter extraction for transmission-line interconnects by using method-of-moments solutions," Microwave and Optical Technology Letters, vol. 31, no. 1, pp. 37-39, Oct. 2001.

[10] J. S. Hong, Y. W. Liu, B. Z. Wang and K. K. Mei, "A novel differential equation model for a microstrip low-pass filter," Microwave and Optical Technology Letters, vol. 35, no. 5, pp. 368-370, Dec. 2002.

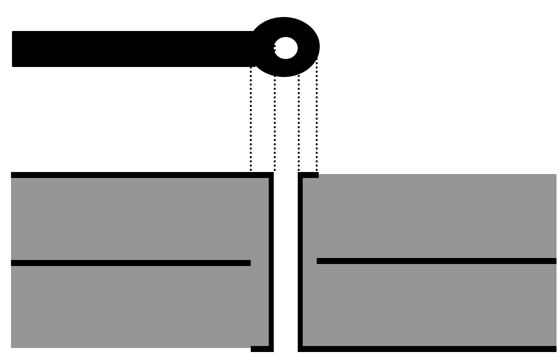

Fig 1. The structure of through hole via embedded in two-layered microstrip circuit.

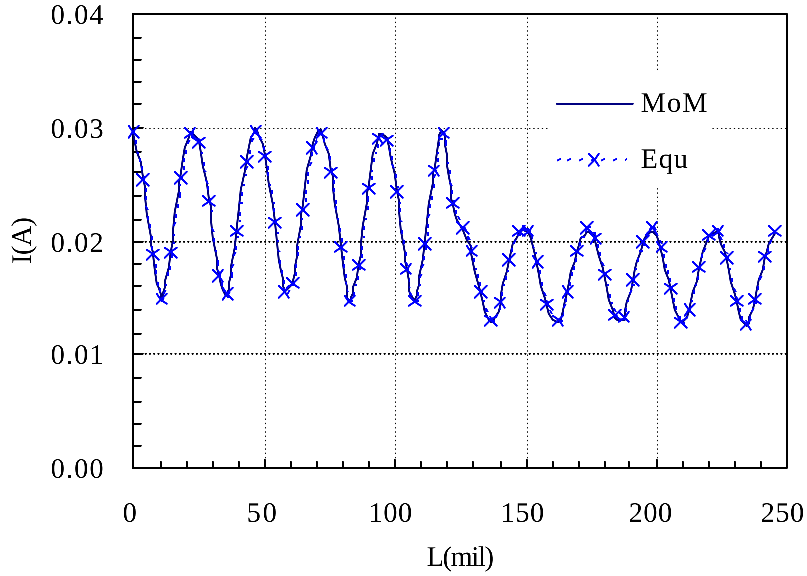

Fig 2. Comparison of on-line current distribution terminated with 20Ω loads between differential equation model (simplified by Equ) and MoM.

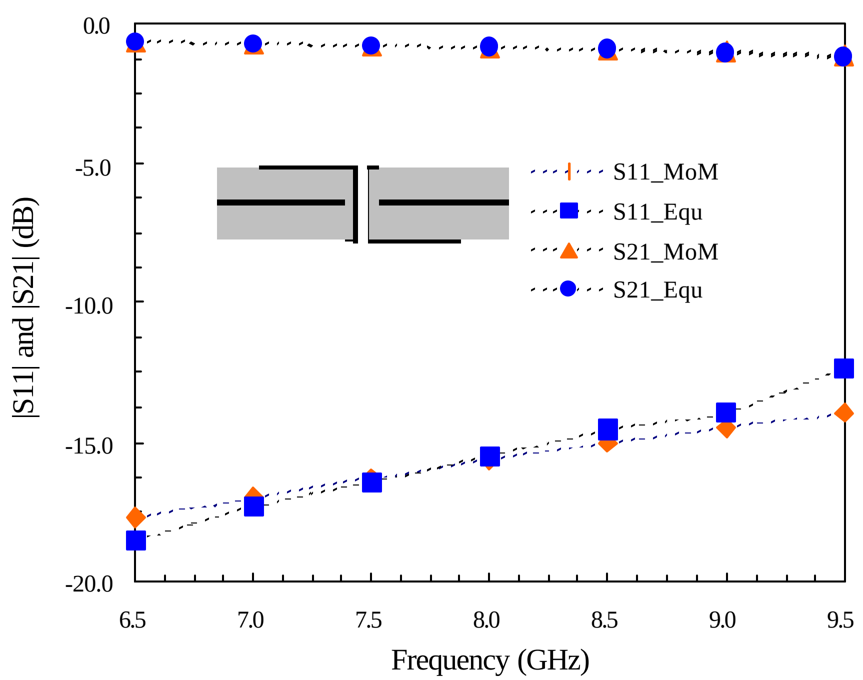

Fig. 3 Comparison of S-parameter at the range from 6.5GHz to 9.5GHz obtained from differential equation model (simplified by Equ) and MoM.